Technical Appendix to ``The Effects of A Money-financed Fiscal Stimulus in A Small Open-economy''

(Not for Publication)

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September 2021

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1 The Model

1.1 Households

Households are line-up [0,1] and openness is v.

(Nominal) Households' budget constraint is given by:

$$P_tC_t + B_t + B_t^*E_t + M_t = B_{t-1}(1+i_{t-1}) + B_{t-1}^*(1+i_{t-1}^*)E_t + M_{t-1} + W_tN_t + D_t - P_tTR_t .$$

Dividing both sides of the previous expression by P_t yields:

$$C_t + \mathrm{B}_t + \frac{E_tP_t^*}{P_t}\mathrm{B}_t^* + L_t = \frac{P_{t-1}}{P_t}\mathrm{B}_{t-1}(1+i_{t-1}) + \frac{E_tP_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}\mathrm{B}_{t-1}^*(1+i_{t-1}^*) + \frac{P_{t-1}}{P_t}L_{t-1} + \frac{W_t}{P}N_t + \frac{D_t}{P_t} - TR_t ,$$

which can be rewritten as:

$$C_t + \mathrm{B}_t + Q_t\mathrm{B}_t^* + L_t = \Pi_t^{-1}\mathrm{B}_{t-1}(1+i_{t-1}) + Q_t(\Pi_t^*)^{-1}\mathrm{B}_{t-1}^*(1+i_{t-1}^*) + \Pi_t^{-1}L_{t-1} + \frac{W_t}{P}N_t + \frac{PR_t}{P_t} - TR_t ,$$

which is identical with Eq.(4) in the text.

with $\mathrm{B}_t \equiv \frac{B_t}{P_t}$, $\mathrm{B}_t^* \equiv \frac{B_t^*}{P_t^*}$ and $Q_t \equiv \frac{E_tP_t^*}{P_t}$.

Alternatively, we have:

$$C_t + \mathrm{B}_t + \frac{E_tP_t^*}{P_t}\mathrm{B}_t^* + L_t = \frac{P_{t-1}}{P_t}\mathrm{B}_{t-1}(1+i_{t-1}) + \frac{E_t}{E_{t-1}}\frac{P_{t-1}}{P_t}\frac{E_{t-1}P_{t-1}^*}{P_{t-1}}\mathrm{B}_{t-1}^*(1+i_{t-1}^*) + \frac{P_{t-1}}{P_t}L_{t-1} + \frac{W_t}{P}N_t + \frac{D_t}{P_t} - TR_t ,$$

which can be rewritten as:

$$C_t + \mathrm{B}_t + Q_t\mathrm{B}_t^* + L_t = \Pi_t^{-1}\mathrm{B}_{t-1}(1+i_{t-1}) + \Pi_t^{-1}\frac{E_t}{E_{t-1}}Q_{t-1}\mathrm{B}_{t-1}^*(1+i_{t-1}^*) + \Pi_t^{-1}L_{t-1} + \frac{W_t}{P_t}N_t + \frac{D_t}{P_t} - TR_t .$$

Further, the previous expression can be rewritten as:

$$C_t + \frac{1}{1+i_t}\left[(1+i_t)\mathrm{B}_t + (1+i_t)Q_t\mathrm{B}_t^* + (1+i_t)L_t\right] = \left[\mathrm{B}_{t-1}(1+i_{t-1}) + \frac{E_t}{E_{t-1}}Q_{t-1}\mathrm{B}_{t-1}^*(1+i_{t-1}^*) + L_{t-1}\right]\Pi_t^{-1} + \frac{W_t}{P_t}N_t + \frac{D_t}{P_t} - TR_t .$$

Further, the previous expression can be rewritten as:

$$C_t+\frac{1}{1+i_t}\left[(1+i_t)\mathrm{B}_t+(1+i_t)Q_t\mathrm{B}_t^*+L_t\right]\Pi_{t+1}^{-1}\Pi_{t+1}+L_t\left(1-\frac{1}{1+i_t}\right)$$
$$=\left[\mathrm{B}_{t-1}(1+i_{t-1})+\frac{E_t}{E_{t-1}}Q_{t-1}\mathrm{B}_{t-1}^*\left(1+i_{t-1}^*\right)+L_{t-1}\right]\Pi_t^{-1}+\frac{W_t}{P_t}N_t+\frac{D_t}{P_t}-TR_t.$$

Let define $\mathrm{A}_t\equiv\left[(1+i_{t-1})\mathrm{B}_{t-1}+Q_{t-1}\mathrm{B}_{t-1}^*\frac{E_t}{E_{t-1}}\left(1+i_{t-1}^*\right)+L_{t-1}\right]\Pi_t^{-1}$. Then, the previous expression can be rewritten as:

$$C_t+\frac{1}{1+i_t}\mathrm{A}_{t+1}\Pi_{t+1}+L_t\left(1-\frac{1}{1+i_t}\right)=\mathrm{A}_t+\frac{W_t}{P_t}N_t+\frac{D_t}{P_t}-TR_t,$$

where we assume that the UIP. The previous expression is identical with Eq.(6) in the text.

Households' optimization problem is given by:

$$\max_{C_t,C_{t+1},N_t,\mathrm{A}_{t+1},L_t}\sum_{t=0}^{\infty}\beta^t U_t,$$

s.t.

$$U_t\equiv\left[U(C_t,L_t)-V(N_t)\right]Z_t,$$

$$C_t+\frac{1}{1+i_t}\mathrm{A}_{t+1}\Pi_{t+1}+L_t\left(1-\frac{1}{1+i_t}\right)=\mathrm{A}_t+\frac{W_t}{P_t}N_t+\frac{D_t}{P_t}-TR_t.$$

The Lagrangean is given by:

$$L\equiv\beta^t\left(U(C_t,L_t)-V(N_t)\right)Z_t+\beta^{t+1}\left(U(C_{t+1},L_{t+1})-V(N_{t+1})\right)Z_{t+1}+\cdots$$
$$+\lambda_t\beta^t\left[\mathrm{A}_t+\frac{W_t}{P_t}N_t+\frac{PR_t}{P_t}-T_t-C_t-\frac{1}{1+i_t}\mathrm{A}_{t+1}\Pi_{t+1}-L_t\left(1-\frac{1}{1+i_t}\right)\right]$$
$$+\lambda_{t+1}\beta^{t+1}\left[\mathrm{A}_{t+1}+\frac{W_{t+1}}{P_{t+1}}N_t+\frac{PR_{t+1}}{P_{t+1}}-T_{t+1}-C_{t+1}-\frac{1}{1+i_{t+1}}\mathrm{A}_{t+2}\Pi_{t+2}-L_{t+1}\left(1-\frac{1}{1+i_{t+1}}\right)\right].$$
$$+\cdots$$

FONCs are given by:

$$\frac{\partial L}{\partial C_t}=\beta^t U_{c,t}Z_t-\beta^t\lambda_t=0,$$

$$\frac{\partial L}{\partial C_{t+1}} = \beta^{t+1} U_{c,t+1} Z_{t+1} - \beta^{t+1} \lambda_{t+1} = 0,$$

$$\frac{\partial L}{\partial N_t} = \beta^t \left(-V_{n,t}\right) Z_t + \lambda_t \beta^t \frac{W_t}{P_t} = 0,$$

$$\frac{\partial L}{\partial \mathrm{A}_{t+1}} = -\lambda_t \beta^t \frac{1}{1+i_t} \Pi_{t+1} + \lambda_{t+1} \beta^{t+1} = 0,$$

$$\frac{\partial L}{\partial L_t} = \beta^t U_{l,t} Z_t - \beta^t \lambda_t \left(1 - \frac{1}{1+i_t}\right) = 0,$$

which can be rewritten as:

$\lambda_t = U_{c,t} Z_t$, (1-1)

$\lambda_{t+1} = U_{c,t+1} Z_{t+1}$, (1-2)

$\lambda_t = \left(\frac{W_t}{P_t}\right)^{-1} V_{n,t} Z_t$, (1-3)

$\lambda_t = \beta \lambda_{t+1} \Pi_{t+1}^{-1} \left(1+i_t\right)$, (1-4)

$\lambda_t = U_{l,t} \left(\frac{i_t}{1+i_t}\right)^{-1} Z_t$, (1-5)

where λ_t denotes Lagrange multiplier.

Combining Eqs.(1-1), (1-2) and (1-4) yields:

$U_{c,t} Z_t = \beta \left(1+i_t\right) \Pi_{t+1}^{-1} U_{c,t+1} Z_{t+1}$. (1-6)

Combing Eqs.(1-1) and (1-3) yields:

$\frac{W_t}{P_t} = \frac{V_{n,t}}{U_{c,t}}$. (1-7)

Combing Eqs.(1-1) and (1-5) yields:

$\frac{U_{l,t}}{U_{c,t}} = \frac{i_t}{1+i_t}$. (1-8)

Note that Eqs.(1-6)—(1-8) are identical with Eqs.(7)—(9) in the text.

1.2 International Risk sharing Condition

Under the assumption of complete markets for securities traded internationally, a condition analogous to Eq.(1-6) must also hold for the representative household in the foreign country:

$U_{c,t}^{*}Z_{t}^{*}=\beta\left(1+i_{t}^{*}\right)\left(\Pi_{t+1}^{*}\right)^{-1}U_{c,t+1}^{*}Z_{t+1}^{*}$. (1-9)

Eqs.(1-6) and (1-9) can be rewritten as:

$$\beta^{-1}=\left(1+i_{t}\right)\Pi_{t+1}^{-1}\frac{U_{c,t+1}}{U_{c,t}}\frac{Z_{t+1}}{Z_{t}},$$

$$\beta^{-1}=\left(1+i_{t}^{*}\right)\left(\Pi_{t+1}^{*}\right)^{-1}\frac{U_{c,t+1}^{*}}{U_{c,t}^{*}}\frac{Z_{t+1}^{*}}{Z_{t}^{*}}.$$

Combining the previous expressions each other yields:

$$\left(1+i_{t}\right)\Pi_{t+1}^{-1}\frac{U_{c,t+1}}{U_{c,t}}\frac{Z_{t+1}}{Z_{t}}=\left(1+i_{t}^{*}\right)\left(\Pi_{t+1}^{*}\right)^{-1}\frac{U_{c,t+1}^{*}}{U_{c,t}^{*}}\frac{Z_{t+1}^{*}}{Z_{t}^{*}}.$$

Dividing both sides of the previous expression by $\left(1+i_{t}\right)\Pi_{t+1}^{-1}$ yields:

$$\frac{U_{c,t+1}}{U_{c,t}}\frac{Z_{t+1}}{Z_{t}}=\frac{1+i_{t}^{*}}{1+i_{t}}\frac{P_{t+1}}{P_{t}}\frac{P_{t}^{*}}{P_{t+1}^{*}}\frac{U_{c,t+1}^{*}}{U_{c,t}^{*}}\frac{Z_{t+1}^{*}}{Z_{t}^{*}}.$$

Plugging the UIP $\frac{1+i_{t}}{1+i_{t}^{*}}=\frac{E_{t+1}}{E_{t}}$ into the previous expression yields:

$$\frac{U_{c,t+1}}{U_{c,t}}\frac{Z_{t+1}}{Z_{t}}=\frac{Q_{t}}{Q_{t+1}}\frac{U_{c,t+1}^{*}}{U_{c,t}^{*}}\frac{Z_{t+1}^{*}}{Z_{t}^{*}}.$$

In the period −1, the previous expression is given by:

$$\frac{U_{c,0}}{U_{c,-1}}\frac{Z_{0}}{Z_{-1}}=\frac{Q_{-1}}{Q_{0}}\frac{U_{c,0}^{*}}{U_{c,-1}^{*}}\frac{Z_{0}^{*}}{Z_{-1}^{*}},$$

which can be rewritten as:

$$U_{c,0}^{-1}=\frac{U_{c,-1}^{*}}{U_{c,-1}}\frac{Z_{-1}^{*}}{Z_{-1}}\frac{1}{Q_{-1}}\left(U_{c,0}^{*}\right)^{-1}Q_{0}\frac{Z_{0}}{Z_{0}^{*}}.$$

Let define $\vartheta\equiv\frac{U_{c,-1}^{*}}{U_{c,-1}}\frac{Z_{-1}^{*}}{Z_{-1}}\frac{1}{Q_{-1}}$ as an initial condition. Then the previous expression can be generalized as follows:

$$U_{c,t}^{-1} = \vartheta \left(U_{c,t}^{*} \right)^{-1} Q_t \frac{Z_t}{Z_t^{*}} \text{, (1-10)}$$

which is identical with Eq.(10) in the text.

1.3 Optimal Allocation of Goods

Let define consumption indices as follows:

$$C_t \equiv \frac{1}{(1-\nu)^{1-\nu} \nu^{\nu}} C_{H,t}^{1-\nu} C_{F,t}^{\nu} \text{, (1-11)}$$

with $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $C_{F,t} \equiv \left[\int_0^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$.

By solving cost-minimization problems for households, we have optimal allocation of expenditures as follows:

$$C_{H,t}(j) = \left(\frac{P_t(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \text{, (1-12)}$$

and

$$C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t} \text{, (1-13)}$$

with:

$$P_{H,t} \equiv \left[\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \text{ and } P_{F,t} \equiv \left[\int_0^1 P_{F,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}.$$

Now, we get total demand for goods produced in the SOE and the ROW. Optimization problem is given by:

$$\max_{C_{H,t}, C_{F,t}} C_t \text{,}$$

s.t.

Eq.(1-11) and $X_t - \left(P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \right) = 0$.

The Lagrangean is given by

$$L \equiv \frac{1}{(1-\nu)^{1-\nu} \nu^{\nu}} C_{H,t}^{1-\nu} C_{F,t}^{\nu} + \lambda \left(X_t - P_{H,t} C_{H,t} - P_{F,t} C_{F,t} \right).$$

The FONCs is given by:

$$\frac{\partial L}{\partial C_{H,t}} = \frac{1}{(1-\nu)^{1-\nu}\nu^{\nu}}(1-\nu)C_{H,t}^{-\nu}C_{F,t}^{\nu} - \lambda P_{H,t}$$
$$= (1-\nu)^{\nu}\nu^{-\nu}C_{H,t}^{-\nu}C_{F,t}^{\nu} - \lambda P_{H,t}$$
$$= 0$$
$$\frac{\partial L}{\partial C_{F,t}} = (1-\nu)^{-(1-\nu)}\nu^{1-\nu}C_{H,t}^{1-\nu}C_{F,t}^{-(1-\nu)} - \lambda P_{F,t}$$
$$= 0$$

These previous expressions can be rewritten as:

$$(1-\nu)^{\nu}\nu^{-\nu}C_{H,t}^{-\nu}C_{F,t}^{\nu} = \lambda P_{H,t},$$

$$(1-\nu)^{-(1-\nu)}\nu^{1-\nu}C_{H,t}^{1-\nu}C_{F,t}^{-(1-\nu)} = \lambda P_{F,t}.$$

Combining these expression yields:

$$(1-\nu)\nu^{-1}C_{H,t}^{-1}C_{F,t} = \frac{P_{H,t}}{P_{F,t}},$$

which can be rewritten as:

$$C_{F,t} = \frac{\nu}{1-\nu}\left(\frac{P_{H,t}}{P_{F,t}}\right)C_{H,t} \quad . \text{(1-14)}$$
$$= \frac{\nu}{1-\nu}S_t^{-1}C_{H,t}$$

Plugging Eq.(1-14) into Eq.(1-11) yields:

$$C_t = \frac{1}{(1-\nu)^{1-\nu}\nu^{\nu}}C_{H,t}^{1-\nu}\left(\frac{\nu}{1-\nu}S_t^{-1}C_{H,t}\right)^{\nu}$$
$$= \frac{\nu^{\nu}}{(1-\nu)^{1-\nu}\nu^{\nu}}(1-\nu)^{-\nu}S_t^{-\nu}C_{H,t} \qquad . \text{(1-15)}$$
$$= \frac{1}{1-\nu}S_t^{-\nu}C_{H,t}$$

The definition of the PPI in the SOE is given by:

$P_t \equiv P_{H,t}^{1-\nu}P_{F,t}^{\nu}$. (1-16)

Eq.(1-15) can be rewritten as:

$C_{H,t} = (1-\nu)S_t^{\nu}C_t$. (1-17) ,

Which is identical with the LHS in Eq.(3) in the text.

Eq.(1-14) can be rewritten as:

$$C_{H,t} = \frac{1-\nu}{\nu} S_t C_{F,t} \,.$$

Plugging the previous expression into Eq.(1-15) yields:

$$\begin{aligned} C_t &\equiv \frac{1}{(1-\nu)^{1-\nu}\nu^{\nu}} \left(\frac{1-\nu}{\nu} S_t C_{F,t} \right)^{1-\nu} C_{F,t}^{\nu} \\ &= \frac{1}{\nu} S_t^{1-\nu} C_{F,t} \end{aligned}$$

which can be rewritten as:

$$C_{F,t} = \nu S_t^{-(1-\nu)} C_t \,, \quad (1\text{-}18)$$

which is identical with the RHS in Eq.(3) in the text.

1.4 Domestic Producers

Production function is given by:

$$Y_t(j) = N_t(j)^{1-\alpha} \,. \quad (1\text{-}19)$$

Combining Eq.(1-19) and $Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$ with the definitions of the PPI indices yields:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} Y_t \,, \quad (1\text{-}20)$$

Plugging Eq.(1-19) into Eq.(1-20) yields:

$$N_t(j)^{1-\alpha} = \left(\frac{P_t(j)}{P_{H,t}} \right)^{-\varepsilon} Y_t \,,$$

which can be rewritten as:

$$N_t(j) = \left(\frac{P_t(j)}{P_{H,t}} \right)^{-\frac{\varepsilon}{1-\alpha}} Y_t^{\frac{1}{1-\alpha}}$$

Let define $N_t \equiv \int_0^1 N_t(j) dj$. Plugging the previous expression into the definition yields:

$$
\begin{aligned}
N_t &= \int_0^1 N_t(j)\,dj \\
&= \int_0^1 \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\frac{\varepsilon}{1-\alpha}} Y_t^{\frac{1}{1-\alpha}} dj\ , \\
&= \int_0^1 \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\frac{\varepsilon}{1-\alpha}} dj Y_t^{\frac{1}{1-\alpha}}
\end{aligned}
$$

The previous expression can be rewritten as:

$N_t^{1-\alpha} = Y_t \Omega^{1-\alpha}$, (1-21)

with $\Omega = \int_0^1 \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\frac{\varepsilon}{1-\alpha}} dj$. Eq.(1-21) is identical with Eq.(14) in the text.

Now we consider firms' maximization problem following Gali (2015). The firms' maximization problem is given by:

$$
\max_{\tilde{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathrm{E}_t \left\{ \Lambda_{t,t+k} \left(\frac{1}{P_{t+k}}\right) \left[\tilde{P}_{H,t} Y_{t+k|t} - \mathbb{C}_{t+k}\left(Y_{t+k|t}\right) \right] \right\},
$$

with $Y_{t+k|t} \equiv \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} Y_{t+k}$ and $\Lambda_{t,t+k} \equiv \mathrm{Q}_{t,t+k}\left(\frac{P_{t+k}}{P_t}\right) = \beta^k \left(\frac{U_{c,t}^{-1}}{U_{c,t+k}^{-1}} \frac{Z_{t+1}}{Z_t}\right)$ being the real stochastic discount factor where $\mathrm{Q}_{t,t+k}$ denotes the price of a one period discount bond paying off one unit of domestic currency. The previous expression can be rewritten as :

$$
\max_{\tilde{P}_{H,t}} \left\{
\begin{aligned}
&\Lambda_{t,t}\left(\frac{1}{P_t}\right)\left[\tilde{P}_{H,t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon} Y_t - \mathbb{C}_t\left(\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon} Y_t\right)\right] + \theta\Lambda_{t,t+1}\left(\frac{1}{P_{t+1}}\right)\left[\tilde{P}_{H,t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t+1}}\right)^{-\varepsilon} Y_{t+1} - \mathbb{C}_{t+1}\left(\left(\frac{\tilde{P}_{H,t}}{P_{H,t+1}}\right)^{-\varepsilon} Y_{t+1}\right)\right] \\
&+\theta^2\Lambda_{t,t+2}\left(\frac{1}{P_{t+2}}\right)\left[\tilde{P}_{H,t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t+2}}\right)^{-\varepsilon} Y_{t+2} - \mathbb{C}_{t+2}\left(\left(\frac{\tilde{P}_{H,t}}{P_{H,t+2}}\right)^{-\varepsilon} Y_{t+2}\right)\right] + \cdots
\end{aligned}
\right.
$$

The FONC for firms is given by:

$$\Lambda_{t,t}\left(\frac{1}{P_t}\right)\left[(1-\varepsilon)\tilde{P}_{H,t}^{-\varepsilon}P_{H,t}^{\varepsilon}Y_t - MC_{t|t}^{n}(-\varepsilon)\tilde{P}_{H,t}^{-\varepsilon-1}P_{H,t}^{\varepsilon}Y_t\right]$$
$$+\theta\Lambda_{t,t+1}\left(\frac{1}{P_{t+1}}\right)\left[(1-\varepsilon)\tilde{P}_{H,t}^{-\varepsilon}P_{H,t+1}^{\varepsilon}Y_{t+1} - MC_{t+1|t}^{n}(-\varepsilon)\tilde{P}_{H,t}^{-\varepsilon-1}P_{H,t+1}^{\varepsilon}Y_{t+1}\right] \quad ,$$
$$+\theta^2\Lambda_{t,t+2}\left(\frac{1}{P_{t+2}}\right)\left[(1-\varepsilon)\tilde{P}_{H,t}^{-\varepsilon}P_{H,t+2}^{\varepsilon}Y_{t+2} - MC_{t+2|t}^{n}(-\varepsilon)\tilde{P}_{H,t}^{-\varepsilon-1}P_{H,t+2}^{\varepsilon}Y_{t+2}\right]+\cdots=0$$

which can be rewritten as:

$$\Lambda_{t,t}\left(\frac{1}{P_t}\right)\left[\tilde{P}_{H,t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon}Y_t - \frac{\varepsilon}{\varepsilon-1}MC_{t|t}^{n}\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon}Y_t\right]$$
$$+\theta\Lambda_{t,t+1}\left(\frac{1}{P_{t+1}}\right)\left[\tilde{P}_{H,t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t+1}}\right)^{-\varepsilon}Y_{t+1} - \frac{\varepsilon}{\varepsilon-1}MC_{t+1|t}^{n}\left(\frac{\tilde{P}_{H,t}}{P_{H,t+1}}\right)^{-\varepsilon}Y_{t+1}\right] \quad ,$$
$$+\theta^2\Lambda_{t,t+2}\left(\frac{1}{P_{t+2}}\right)\left[\tilde{P}_{H,t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t+2}}\right)^{-\varepsilon}Y_{t+2} - \frac{\varepsilon}{\varepsilon-1}MC_{t+2|t}^{n}\left(\frac{\tilde{P}_{H,t}}{P_{H,t+2}}\right)^{-\varepsilon}Y_{t+2}\right]+\cdots=0$$

with $MC_{t+k|t}^{n} \equiv \mathbb{C}'_{t+k}\left(Y_{t+k|t}\right)$ being the nominal marginal cost.

By using the definition $Y_{t+k|t} \equiv \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} Y_{t+k}$, the previous expression can be rewritten as:

$$\Lambda_{t,t}\left(\frac{1}{P_t}\right)\left[\tilde{P}_{H,t}Y_{t|t} - \frac{\varepsilon}{\varepsilon-1}MC_{t|t}^{n}Y_{t|t}\right]$$
$$+\theta\Lambda_{t,t+1}\left(\frac{1}{P_{t+1}}\right)\left[\tilde{P}_{H,t}Y_{t+1|t} - \frac{\varepsilon}{\varepsilon-1}MC_{t+1|t}^{n}Y_{t+1|t}\right] \quad ,$$
$$+\theta^2\Lambda_{t,t+2}\left(\frac{1}{P_{t+2}}\right)\left[\tilde{P}_{H,t}Y_{t+2|t} - \frac{\varepsilon}{\varepsilon-1}MC_{t+2|t}^{n}Y_{t+2|t}\right]+\cdots=0$$

which can be rewritten as:

$$\Lambda_{t,t}\left(\frac{1}{P_t}\right)Y_{t|t}\left(\tilde{P}_{H,t}-\frac{\varepsilon}{\varepsilon-1}MC^n_{t|t}\right)$$
$$+\theta\Lambda_{t,t+1}\left(\frac{1}{P_{t+1}}\right)Y_{t+1|t}\left(\tilde{P}_{H,t}-\frac{\varepsilon}{\varepsilon-1}MC^n_{t+1|t}\right) \quad .$$
$$+\theta^2\Lambda_{t,t+2}\left(\frac{1}{P_{t+2}}\right)Y_{t+2|t}\left(\tilde{P}_{H,t}-\frac{\varepsilon}{\varepsilon-1}MC^n_{t+2|t}\right)+\cdots=0$$

The previous expression can be compact expression as:

$$\sum_{k=0}^{\infty}\theta^k \mathrm{E}_t\left[\Lambda_{t,t+k}\left(\frac{1}{P_{t+k}}\right)Y_{t+k|t}\left(\tilde{P}_{H,t}-\frac{\varepsilon}{\varepsilon-1}MC^n_{t+k|t}\right)\right]=0\text{, (1-22)}$$

which is identical with Eq.(15) in the text.

Nominal marginal cost is given by:

$$MC^n_{t+k|t}=\frac{W_{t+k}}{MPN_{t+k|t}}\text{,}$$

with $MPN_{t+k}\equiv\frac{\partial Y_{t+k}}{\partial N_{t+k}}$ and $MPN_{t+k|t}\equiv\frac{\partial Y_{t+k}}{\partial N_{t+k|t}}$.

Plugging the definition of the real marginal cost into Eq.(1-22) yields:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left[\left(\frac{U^{-1}_{c,t}}{U^{-1}_{c,t+k}}\frac{Z_{t+1}}{Z_t}\right)\left(\frac{1}{P_{t+k}}\right)Y_{t+k|t}\left(\tilde{P}_{H,t}-\frac{\varepsilon}{\varepsilon-1}MC^n_{t+k|t}\right)\right]=0\text{ .}$$

By multiplying $U_{c,t}Z_t$ both sides of the previous expression yields:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left[\left(\frac{1}{P_{t+k}U^{-1}_{c,t+k}}\right)Z_{t+1}Y_{t+k|t}\left(\tilde{P}_{H,t}-\frac{\varepsilon}{\varepsilon-1}MC^n_{t+k|t}\right)\right]=0\text{,}$$

which can be rewritten as:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left[\left(\frac{1}{P_{t+k}U^{-1}_{c,t+k}}\right)Z_{t+1}Y_{t+k|t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t-1}}-\frac{\varepsilon}{\varepsilon-1}\frac{MC^n_{t+k|t}}{P_{H,t+k}}\frac{P_{H,t+k}}{P_{H,t-1}}\right)\right]=0\text{.}$$

Let define $MC_{t+k|t}\equiv\frac{MC^n_{t+k|t}}{P_{H,t+k}}$ and $\Pi_{H,t-1,t+k}\equiv\frac{P_{H,t+k}}{P_{H,t-1}}$. Then, the previous expression can be rewritten as:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left[\left(\frac{1}{P_{t+k}U^{-1}_{c,t+k}}\right)Z_{t+1}Y_{t+k|t}\left(\frac{\tilde{P}_{H,t}}{P_{H,t-1}}-\frac{\varepsilon}{\varepsilon-1}MC_{t+k|t}\Pi_{H,t-1,t+k}\right)\right]=0\text{ .}$$

Let define $\tilde{X}_{H,t} \equiv \frac{\tilde{P}_{H,t}}{P_{H,t-1}}$. Then the previous expression can be rewritten as:

$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\left(U_{c,t+k}^{-1}\right)^{-1} Z_{t+k}\right] Y_{t+k|t} \frac{P_{H,t-1}}{P_{t+k}}\left(\tilde{X}_{H,t}-\frac{\varepsilon}{\varepsilon-1}\Pi_{H,t-1,t+k} MC_{t+k|t}\right)\right\}=0$. Note that $P_{H,t-1}$ is multiplied on both sides of the previous expression. The previous expression can be rewritten as:

which can be rewritten as:

$$
\begin{aligned}
&\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} \frac{P_{H,t}}{P_t}\frac{P_{H,t-1}}{P_{H,t}}\left(\tilde{X}_{H,t}-\frac{\varepsilon}{\varepsilon-1}\frac{P_{H,t}}{P_{H,t-1}} MC_{t|t}\right)\\
&+\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} \frac{P_{H,t+1}}{P_{t+1}}\frac{P_{H,t}}{P_{H,t+1}}\frac{P_{H,t-1}}{P_{H,t}}\left(\tilde{X}_{H,t}-\frac{\varepsilon}{\varepsilon-1}\frac{P_{H,t+1}}{P_{H,t}}\frac{P_{H,t}}{P_{H,t-1}} MC_{t+1|t}\right)\\
&+(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} \frac{P_{H,t+2}}{P_{t+2}}\frac{P_{H,t+1}}{P_{H,t+2}}\frac{P_{H,t}}{P_{H,t+1}}\frac{P_{H,t-1}}{P_{H,t}}\left(\tilde{X}_{H,t}-\frac{\varepsilon}{\varepsilon-1}\frac{P_{H,t+2}}{P_{H,t+1}}\frac{P_{H,t+1}}{P_{H,t}}\frac{P_{H,t}}{P_{H,t-1}} MC_{t+2|t}\right)\\
&+\cdots=0
\end{aligned},
$$

which can be rewritten as:

$$
\begin{aligned}
&\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} g\left(S_t\right)^{-1}\Pi_{H,t}^{-1}\left(\tilde{X}_{H,t}-\frac{\varepsilon}{\varepsilon-1}\Pi_{H,t} MC_{t|t}\right)\\
&+\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} g\left(S_{t+1}\right)^{-1}\Pi_{H,t+1}^{-1}\Pi_{H,t}^{-1}\left(\tilde{X}_{H,t}-\frac{\varepsilon}{\varepsilon-1}\Pi_{H,t+1}\Pi_{H,t} MC_{t+1|t}\right)\\
&+(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} g\left(S_{t+2}\right)^{-1}\Pi_{H,t+2}^{-1}\Pi_{H,t+1}^{-1}\Pi_{H,t}^{-1}\left(\begin{array}{l}\tilde{X}_{H,t}\\-\frac{\varepsilon}{\varepsilon-1}\Pi_{H,t+2}\Pi_{H,t+1}\Pi_{H,t} MC_{t+2|t}\end{array}\right)\\
&+\cdots=0
\end{aligned},
$$

with $g\left(S_t\right) \equiv \frac{P_t}{P_{H,t}}=\frac{P_{H,t}^{1-\nu}P_{F,t}^{\nu}}{P_{H,t}}=\left(\frac{P_{F,t}}{P_{H,t}}\right)^{\nu}=S_t^{\nu}$. The previous expression can be rewritten as:

$$
\begin{aligned}
&\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} S_t^{-\nu} \Pi_{H,t}^{-1} \tilde{X}_{H,t} - \frac{\varepsilon}{\varepsilon-1}\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} S_t^{-\nu} MC_{t|t} \\
&+\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} S_{t+1}^{-\nu} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} \tilde{X}_{H,t} \\
&-\frac{\varepsilon}{\varepsilon-1}\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} S_{t+1}^{-\nu} MC_{t+1|t} \\
&+(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} S_{t+2}^{-\nu} \Pi_{H,t+2}^{-1} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} \tilde{X}_{H,t} \\
&-\frac{\varepsilon}{\varepsilon-1}(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} S_{t+2}^{-\nu} MC_{t+2|t} \\
&+\cdots = 0
\end{aligned}
.
$$

By moving the terms related to the marginal cost to the RHS yields:

$$
\begin{aligned}
&\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} S_t^{-\nu} \Pi_{H,t}^{-1} \tilde{X}_{H,t} \\
&+\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} S_{t+1}^{-\nu} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} \tilde{X}_{H,t} \\
&+(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} S_{t+2}^{-\nu} \Pi_{H,t+2}^{-1} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} \tilde{X}_{H,t} \\
&+\cdots \\
= \;&\frac{\varepsilon}{\varepsilon-1}\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} S_t^{-\nu} MC_{t|t} \\
&+\frac{\varepsilon}{\varepsilon-1}\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} S_{t+1}^{-\nu} MC_{t+1|t} \\
&+\frac{\varepsilon}{\varepsilon-1}(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} S_{t+2}^{-\nu} MC_{t+2|t} \\
&+\cdots
\end{aligned}
$$

which can be simplified as follows:

$$
\begin{aligned}
&\tilde{X}_{H,t}\left\{
\begin{aligned}
&\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} S_t^{-\nu} \Pi_{H,t}^{-1} \\
&+\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} S_{t+1}^{-\nu} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} \\
&+(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} S_{t+2}^{-\nu} \Pi_{H,t+2}^{-1} \Pi_{H,t+1}^{-1} \Pi_{H,t}^{-1} + \cdots
\end{aligned}
\right\} \\
&= \frac{\varepsilon}{\varepsilon-1}\left\{
\begin{aligned}
&\left[\left(U_{c,t}^{-1}\right)^{-1} Z_t\right] Y_{t|t} S_t^{-\nu} MC_{t|t} \\
&+\theta\beta\left[\left(U_{c,t+1}^{-1}\right)^{-1} Z_{t+1}\right] Y_{t+1|t} S_{t+1}^{-\nu} MC_{t+1|t} \\
&+(\theta\beta)^2\left[\left(U_{c,t+2}^{-1}\right)^{-1} Z_{t+2}\right] Y_{t+2|t} S_{t+2}^{-\nu} MC_{t+2|t} + \cdots
\end{aligned}
\right\}
\end{aligned}
.
$$

Then, we have:

$$\tilde{X}_{H,t}\sum_{k=0}^{\infty}(\theta\beta)^k\left[\left(U_{c,t+k}^{-1}\right)^{-1}Z_{t+k}\right]Y_{t+k|t}S_{t+k}^{-\nu}\prod_{h=0}^{k}\Pi_{H,t+h}^{-1}=\frac{\varepsilon}{\varepsilon-1}\sum_{k=0}^{\infty}(\theta\beta)^k\left[\left(U_{c,t+k}^{-1}\right)^{-1}Z_{t+k}\right]Y_{t+k|t}S_{t+k}^{-\nu}MC_{t+k|t},$$

or:

$$\tilde{X}_{H,t}=\frac{\frac{\varepsilon}{\varepsilon-1}\sum_{k=0}^{\infty}(\theta\beta)^k\left[\left(U_{c,t+k}^{-1}\right)^{-1}Z_{t+k}\right]Y_{t+k|t}g\left(S_{t+k}\right)^{-1}MC_{t+k|t}}{\sum_{k=0}^{\infty}(\theta\beta)^k\left[\left(U_{c,t+k}^{-1}\right)^{-1}Z_{t+k}\right]Y_{t+k|t}g\left(S_{t+k}\right)^{-1}\prod_{h=0}^{k}\Pi_{H,t+h}^{-1}}. \quad (1\text{-}23)$$

Plugging

$$\begin{aligned}MC_{t+k|t}^{n}&=\frac{W_{t+k}}{MPN_{t+k|t}}\\&=W_{t+k}\left(\frac{\partial Y_{t+k}}{\partial N_{t+k|t}}\right)^{-1}\\&=W_{t+k}\left[(1-\alpha)N_{t+k|t}^{-\alpha}\right]^{-1}\\&=\frac{W_{t+k}}{1-\alpha}N_{t+k|t}^{\alpha}\end{aligned},$$

into the definition of the marginal cost yields:

$$MC_{t+k}\equiv\frac{W_{t+k}}{P_{H,t+k}(1-\alpha)}N_{t+k}^{\alpha}. \quad (1\text{-}24)$$

1.5 Market Clearing Condition

The market clearing conditions in the SOE and the ROW are given by:

$$Y_t(j)=C_{H,t}(j)+EX_t(j)+G_t(j), \quad (1\text{-}25)$$

where $EX_t(j)$ is export demand for the good produced by firm *j* and $G_t(j)$ denotes the government purchase for the good produced by firm *j*, respectively.

Combining $G_t\equiv\left[\int_0^1 G_t(j)^{\frac{\varepsilon-1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$ with the definitions of the PPIs yields:

$$G_t(j)=\left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon}G_{H,t}. \quad (1\text{-}26)$$

Similar to Eq.(1-12), we assume that the demands for $C_{H,t}^{*}(j)$ follows as:

$$EX_t(j) = \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\varepsilon} EX_t \text{ , (1-27)}$$

Analogous to Eq.(1-18), demands for domestic goods in foreign country is assumed as:

$$\begin{aligned} EX_t &= \nu \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-1} Y^*_t \\ &= \nu \left(\frac{P^*_{H,t}}{P^*_{F,t}}\right)^{-1} Y^*_t \text{ , (1-28)} \\ &= \nu S_t Y^*_t \end{aligned}$$

which is identical with Eq.(17) in the text.

Plugging Eqs. (1-12), (1-20), (1-26) and (1-27) into Eq.(1-25) yields:

$$\begin{aligned} \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} Y_t &= \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} + \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\varepsilon} EX_t + \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon} G_t \\ &= \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon} \left(C_{H,t} + EX_t + G_t\right) \end{aligned},$$

where we use the LOOP implying that $P^*_{H,t}(j) = \frac{P_{H,t}(j)}{E_t}$ and $P^*_{H,t} = \frac{P_{H,t}}{E_t}$. By dividing both sides of the previous expression $\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}$ yields as follows:

$$Y_t = C_{H,t} + EX_t + G_t .$$

Plugging Eqs.(1-17) and (1-28) into the previous expression yields:

$$Y_t = (1-\nu) S^{\nu}_t C_t + \nu S_t Y^*_t + G_t \text{ , (1-29)}$$

where we apply the definition of the TOT $S_t \equiv \frac{P_{F,t}}{P_{H,t}}$ and $Y^*_t = C^*_t$. Eq.(1-29) is identical with Eq(21) in the text.

1.6 Government Budget Constraint

The government budget constraint is given by:

$$P_{H,t} G_t + B_{t-1}(1 + i_{t-1}) = P_t TR_t + B_t + \Delta M_t \text{ , (1-30)}$$

which is identical with Eq.(18) in the text.

Dividing both side of Eq.(1-30) by P_t yields:

$$\frac{P_{H,t}}{P_t}G_t + \mathrm{B}_{t-1}\left(1+i_{t-1}\right)\frac{P_{t-1}}{P_t} = TR_t + \mathrm{B}_t + \frac{\Delta M_t}{P_t}.$$

Let define the (ex-post) real interest rate $\mathrm{P}_t \equiv \left(1+i_t\right)\frac{P_t}{P_{t+1}}$. Then the previous expression can be rewritten as:

$$S_t^{-\nu}G_t + \mathrm{B}_{t-1}\mathrm{P}_{t-1} = T_t + \mathrm{B}_t + \frac{\Delta M_t}{P_t}, \quad (1\text{-}31)$$

where we use $\frac{P_{H,t}}{P_t} = \frac{P_{H,t}}{P_{H,t}^{1-\nu}P_{F,t}^{\nu}} = S_t^{-\nu}$. Eq(3-1) is identical with Eq.(20) in the text.

The level of seignorage, expressed as a fraction of steady state output can be approximated as:

$$\begin{aligned}\frac{\Delta M_t}{P_t}\frac{1}{Y} &= \frac{\Delta M_t}{P_t}\frac{M_{t-1}}{M_{t-1}}\frac{P_{t-1}}{P_{t-1}}\frac{1}{Y} \\ &= \frac{\Delta M_t}{M_{t-1}}\frac{M_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}\frac{1}{Y}. \quad (1\text{-}32) \\ &= \frac{\Delta M_t}{M_{t-1}}\frac{P_{t-1}}{P_t}L_{t-1}\frac{1}{Y}\end{aligned}$$

Quantity theory of money implies as follows:

$$MV = PY,$$

which can be rewritten as:

$$V^{-1} = \frac{L}{Y}.$$

Plugging the previous expression into Eq.(1-32) yields:

$$\frac{\Delta M_t}{P_t}\frac{1}{Y} = \chi \Delta m_t, \quad (1\text{-}33)$$

with $\chi \equiv V^{-1}$ being the inverse of income velocity of money. Note that Eq.(1-33) ignore changes in the inflation and the deviation of the real money balance from its steady state.

If we do not ignore them, we have:

$$\begin{aligned}\frac{\Delta M_t}{P_t}\frac{1}{Y} &= \frac{\Delta M_t}{M_{t-1}}\frac{P_{t-1}}{P_t}L_{t-1}\frac{1}{Y} \\ &= \ln\left(\frac{M_t}{M_{t-1}}\right)\Pi_{t-1}^{-1}\frac{L_{t-1}}{L}\frac{L}{Y} \\ &= \chi \ln\left(\frac{M_t}{M_{t-1}}\right)\Pi_{t-1}^{-1}\frac{L_{t-1}}{L}\end{aligned}$$

1.7 Trade Balance

Similar to Gali and Monaceli (2005, RES), We define the real trade balance as follows:

$$\begin{aligned}\frac{NX_t}{P_{H,t}} &= Y_t - g(S_t)C_t - G_t \\ &= Y_t - S_t^{\nu} C_t - G_t\end{aligned}, \text{ (1-34)}$$

with NX_t being the (nominal) trade balance. Eq.(34) is identical with Eq.(22) in the text.

Note that:

$$g(S_t) = \frac{P_t}{P_{H,t}} = \frac{P_{H,t}^{1-\nu} P_{F,t}^{\nu}}{P_{H,t}} = S_t^{\nu}.$$

2 The Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_{H,t} = \Pi_t = 1$. Further, we assume $Z_t = Z_t^* = 1$ and $G_t = 0$.

Eqs.(1-6) and (1-9) implies as follows:

$$\begin{aligned}\beta &= \frac{1}{1+i} \\ &= \frac{1}{1+i^*}\end{aligned}. \text{ (2-1)}$$

Eq.(1-7) implies that:

$$\frac{W}{P} = \frac{V_n}{U_c}. \text{ (2-2)}$$

Eq.(1-8) implies as follows:

$$\frac{U_l}{U_c} = \beta i. \text{ (2-3)}$$

Eq.(1-23) implies:

$$1 = \frac{\frac{\varepsilon}{\varepsilon-1}\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]\left[\left(U_c^{-1}\right)^{-1}\right] Y g(S)^{-1} MC}{\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]\left[\left(U_c^{-1}\right)^{-1}\right] Y g(S)^{-1}},$$

which can be rewritten as:

$MC = \mathrm{M}^{-1}$, (2-4)

with $\mathrm{M} \equiv \frac{\varepsilon}{\varepsilon - 1}$ being the constant markup.

Eq.(1-24) implies:

$MC = \frac{1}{1-\alpha} \frac{W}{P_H} N^{\alpha}$. (2-5).

Eq.(1-16) can be rewritten as:

$\frac{V_n}{U_c} = \frac{W}{P_H} \frac{P_H}{P}$. (2-6)

Plugging Eq.(2-5) into Eq.(2-6) yields:

$\frac{V_n}{U_c} = \frac{1-\alpha}{N^{\alpha}\mathrm{M}} \frac{P_H}{P}$. (2-7)

Let define:

$g(S) \equiv \frac{P}{P_H} = S^{\nu}$. (2-8)

Plugging Eq.(2-8) into Eq.(2-7) yields:

$\frac{V_n}{U_c} = \frac{1-\alpha}{N^{\alpha}\mathrm{M}S^{\nu}}$,

which can be written as:

$V_n = \frac{1-\alpha}{N^{\alpha}\mathrm{M}S^{\nu}} U_c$. (2-9)

Eq.(1-10) implies:

$U_c^{-1} = \vartheta \left(U_c^*\right)^{-1} q(S)$, (2-10)

with $q(S) \equiv Q$. Note that:

$Q = \frac{EP^*}{P} = \frac{EP_F^*}{P_H^{1-\nu} P_F^{\nu}} = \frac{P_F}{P_H^{1-\nu} P_F^{\nu}} = \left(\frac{P_F}{P_H}\right)^{1-\nu} = S^{1-\nu}$. (2-11)

Eq.(2-10) can be rewritten as:

$U_c^{-1} = \vartheta \left(U_c^*\right)^{-1} S^{1-\nu}$,

where we use Eq.(2-11). The previous expression can be rewritten as:

$S^{\nu} = \vartheta \left(U_c^*\right)^{-1} S U_c$.

Plugging the previous expression into Eq.(2-9) yields:

$$V_n = \frac{1-\alpha}{N^{\alpha}} \frac{\mathrm{M}^{-1}}{S\vartheta\left(U_c^*\right)^{-1}}. \text{ (2-12)}$$

Let define $H\left(S,U_c^*\right) \equiv V_n N^{\alpha}$. Plugging this definition into Eq.(2-12) yields:

$$H\left(S,U_c^*\right) \equiv (1-\alpha) \frac{\mathrm{M}^{-1}}{S\vartheta\left(U_c^*\right)^{-1}}.$$

Notice that $H_S < 0$, $\lim_{S\to 0} H\left(S,U_c^*\right) = +\infty$ and $\lim_{S\to\infty} H\left(S,U_c^*\right) = 0$.

On the other hand, the market clearing Eq.(1-29) implies:

$$Y = (1-\nu)S^{\nu}C + \nu SY^*. \text{ (2-13)}$$

Because of $C = F\left(U_c^{-1}\right)$ and Eq.(2-10), we have:

$$\begin{aligned} C &= F\left[\vartheta\left(U_c^*\right)^{-1} q(S)\right] \\ &= F\left[\vartheta\left(U_c^*\right)^{-1} S^{1-\nu}\right], \end{aligned}$$

with F being the operator of function.
Plugging the previous expression into Eq.(2-13) yields:

$$Y = (1-\nu)S^{\nu}F\left[\vartheta\left(U_c^*\right)^{-1} S^{1-\nu}\right] + \nu SC^*. \text{ (2-14)}$$

Let define $J\left(S,C^*\right) \equiv (1-\nu)S^{\nu}F\left[\vartheta\left(U_c^*\right)^{-1} S^{1-\nu}\right] + \nu SY^*$. Note that $J_S > 0$, $\lim_{S\to 0} J\left(S,C^*\right) = 0$ and $\lim_{S\to\infty} J\left(S,C^*\right) = +\infty$.

Hence, given a value for C^*, ϑ and Y^*, Eqs.(2-12) and (2-14), jointly determine the steady state value for S and $q(S)$, i.e., the steady state value of the TOT and the real exchange rate (Figure TA-1). This way to show how the TOT as well as the real exchange rate is pinned down in the steady state is almost same as Gali and Monacelli (2002, NBER-WP).
Dividing both sides of Eq.(2-13) by C^* yields:

$$\frac{Y}{C^*} = (1-\nu)S^{\nu}\frac{C}{C^*} + \nu S.$$

For convenience, and without loss of generality, we can assume that initial conditions (i.e., initial distribution of wealth) are such that $\vartheta = 1$ which implies that $Q = \frac{C}{C^*}$.

Plugging this condition into the previous expression yields:

$$\begin{aligned}\frac{Y}{C^*} &= (1-\nu)S^{\nu}Q + \nu S \\ &= (1-\nu)S^{\nu}S^{1-\nu} + \nu S, \\ &= S\end{aligned}$$

which can be rewritten as:

$Y = SY^*$, (2-15)

by using $Y^* = C^*$ which is the steady state market clearing condition in the foreign country.

Let assume symmetric labor market in the foreign country. Then, following condition is applicable:

$$\frac{V_n^*}{U_c^*} = \frac{1-\alpha}{\left(N^*\right)^{\alpha}\mathrm{M}}, \text{ (2-16)}$$

similar to Eq.(2-7).

Dividing Eq.(2-16) by Eq.(2-9) yields:

$$\frac{V_n^*}{V_n}\left(\frac{N^*}{N}\right)^{\alpha} = \frac{U_c^*}{U_c}S^{\nu}. \text{ (2-17)}$$

Combining Eq.(2-10) and the initial condition yields:

$\frac{U_c^*}{U_c} = S^{1-\nu}$, (2-18)

where we use $Q = S^{1-\nu}$. Plugging Eq.(2-18) into Eq.(2-17) yields:

$$\frac{V_n^*}{V_n}\left(\frac{N^*}{N}\right)^{\alpha} = S. \text{ (2-19)}$$

In the steady state, Eq.(14) in the text implies as follows:

$N^{1-\alpha} = Y$.

Plugging the previous expression into Eq.(2-19) yields:

$$\frac{V_n^*}{V_n}\left(\frac{Y^*}{Y}\right)^{\frac{\alpha}{1-\alpha}} = S,$$

where we use the foreign country has production technology identical to Eq.(14).

Plugging Eq.(2-15) into the previous expression yields:

$$\frac{V_n^*}{V_n} = S^{\frac{1}{1-\alpha}}.$$

Let multiply $\left(U_c^*\right)^{-1}$ on both sides of the previous expression. Then, we have:

$$\frac{V_n^*}{U_c^*} = S^{\frac{1-(1-\alpha)(1-\nu)}{1-\alpha}} \frac{V_n}{U_c}, \text{ (2-20)}$$

where we use Eq.(2-18). Let define $\frac{V_n}{U_c} \equiv 1-\Phi$ where Φ denotes the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor (See Benigno and Woodford, 2005). We assume this steady state wedge is common throughout the world, i.e., $\frac{V_n}{U_c} \equiv 1-\Phi = \frac{V_n^*}{U_c^*}$.

Then, Eq.(2-20) boils down to:

$S = 1$, (2-21)

which implies that the PPP is applicable in the long run.

Plugging Eq.(2-21) into Eq.(2-15) yields:

$Y = Y^*$. (2-22)

Plugging Eq.(2-21) into the initial condition yields:

$C = C^*$.

Combining the previous expression, the steady state market clearing condition in the foreign country $Y^* = C^*$ and Eq.(2-22) yields:

$Y = C$.

Finally, Eq.(1-22) implies:

$$P = \frac{\varepsilon}{\varepsilon-1} MC^n, \text{ (2-16)}$$

which can be rewritten as:

$$\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-1} = MC,$$

which corresponds to Eq.(2-4).

3 Log-linearization of the Model

3.1 Log-linearizing the International Risk Sharing Condition

By raising both sides of Eq.(1-10) to the power of −1, we get:

$$U_{c,t} = U^*_{c,t} Q_t^{-1} \vartheta^{-1} \frac{Z^*_t}{Z_t}.$$

Total derivative of the previous expression yields:

$$dU_{c,t} = \vartheta^{-1} dU^*_{c,t} - \vartheta^{-1} U^*_c dQ_t + \vartheta^{-1} U^*_c dZ^*_t - \vartheta^{-1} U^*_c dZ_t.$$

$$\begin{aligned} dQ_t &= -\vartheta^{-1} U^*_c U_c^{-2} U_{cc} dC_t + \vartheta^{-1} U_c^{-1} U^*_{cc} dC^*_t + U_c^{-1} U^*_c \vartheta^{-1} dZ^*_t + U_c^{-1} U^*_c \vartheta^{-1} (-1) dZ_t \\ &= -\vartheta^{-1} \frac{U^*_c}{U_c} \frac{U_{cc}}{U_c} C \frac{dC_t}{C} + \vartheta^{-1} \frac{U^*_c}{U_c} \frac{U^*_{cc}}{U^*_c} C^* \frac{dC^*_t}{C^*} + \vartheta^{-1} \frac{U^*_c}{U_c} dZ^*_t - \vartheta^{-1} \frac{U^*_c}{U_c} dZ_t \\ &= \vartheta^{-1} \frac{U^*_c}{U_c} \left(-\frac{U_{cc}}{U_c} C \right) \frac{dC_t}{C} - \vartheta^{-1} \frac{U^*_c}{U_c} \left(-\frac{U^*_{cc}}{U^*_c} C^* \right) \frac{dC^*_t}{C^*} + \vartheta^{-1} \frac{U^*_c}{U_c} dZ^*_t - \vartheta^{-1} \frac{U^*_c}{U_c} dZ_t \end{aligned}.$$

Dividing both sides of the previous expression yields:

$$\log\left(\frac{U_{c,t}}{U_c}\right) = \log\left(\frac{U^*_{c,t}}{U^*_c}\right) - \log Q_t - \left[\log\left(\frac{Z^*_t}{Z_t}\right)\right].$$

The previous expression can be rewritten as:

$\hat{\xi}_t = -q_t + \hat{\xi}^*_t - \zeta_t$, (3-1)

where $q_t \equiv \log Q_t$, $\hat{\xi}_t \equiv \log\left(\frac{U_{c,t}}{U_c}\right)$, $\hat{\xi}^*_t \equiv \log\left(\frac{U^*_{c,t}}{U^*_c}\right)$ and $\zeta_t \equiv -\log\left(\frac{Z^*_t}{Z_t}\right)$.

Total derivative of the definition of the real exchange rate is given by:

$$dQ_t = \frac{dP_{F,t}}{P_F} - \frac{dP_t}{P},$$

which can be rewritten as:

$q_t = p_{F,t} - p_t$ (3-2).

Dividing both sides of the previous expression by *P* yields:

$$\log\left(\frac{P_t}{P}\right) = (1-\nu)\log\left(\frac{P_{H,t}}{P_H}\right) + \nu\log\left(\frac{P_{F,t}}{P_F}\right),$$

which can be rewritten as:

$p_t = (1-\nu) p_{H,t} + \nu p_{F,t}$. (3-3)

Plugging Eq.(3-3) into Eq.(3-2) yields:

$$\begin{aligned} q_t &= p_{F,t} - \left[(1-\nu)p_{H,t} + \nu p_{F,t}\right] \\ &= (1-\nu)\left(p_{F,t} - p_{H,t}\right) \end{aligned}. \text{ (3-4)}$$

Total derivative of the definition of the TOT is given by:

$$\begin{aligned} dS_t &= \frac{dP_{F,t}}{P_H} - P_F P_H^{-2} dP_{H,t} \\ &= \frac{dP_{F,t}}{P_F} - \frac{dP_{H,t}}{P_H} \end{aligned},$$

which can be rewritten as:

$$\log S_t = \log\left(\frac{P_{F,t}}{P_F}\right) - \log\left(\frac{P_{H,t}}{P_H}\right).$$

Then, we have:

$s_t = p_{F,t} - p_{H,t}$. (3-5)

Plugging Eq.(3-5) into Eq.(3-4) yields:

$q_t = (1-\nu)s_t$. (3-6)

Plugging Eq.(3-6) into Eq.(3-1) yields:

$\hat{\xi}_t = -(1-\nu)s_t + \hat{\xi}_t^* - \zeta_t$, (3-7)

which is identical with Eq.(23) in the text.

3.2 Log-linearizing the Market Clearing Condition

Eq.(1-29) can be rewritten as:

$Y_t = (1-\nu)S_t^{\nu}C_t + \nu S_t C_t^* + G_t$, (3-8)

Total derivative of Eq.(3-8) yields:

$dY_t = \left[(1-\nu)C\nu + \nu C^*\right]dS_t + (1-\nu)dC_t + \nu dC_t^* + dG_t$. (3-9)

By dividing both sides of Eq.(3-9) by Y yields:

$$\begin{aligned} \log\left(\frac{Y_t}{Y}\right) &= \left[(1-\nu)\nu + \nu\right]\log S_t + (1-\nu)\log\left(\frac{C_t}{C}\right) + \nu\log\left(\frac{C_t^*}{C}\right) + \log\left(\frac{G_t}{Y}\right) \\ &= \nu(2-\nu)\log S_t + (1-\nu)\log\left(\frac{C_t}{C}\right) + \nu\log\left(\frac{C_t^*}{C}\right) + \log\left(\frac{G_t}{Y}\right) \end{aligned},$$

where we use the fact that $Y = C = Y^* = C^*$. The previous expression can be rewritten as:

$\hat{y}_t = \nu(2-\nu)s_t + (1-\nu)\hat{c}_t + \nu\hat{y}_t^* + \hat{g}_t$, (3-10)

with $\hat{c}_t \equiv \log\left(\frac{C_t}{C}\right)$, $\hat{y}_t^* \equiv \log\left(\frac{Y_t^*}{Y^*}\right)$ where we use $\hat{y}_t^* = \hat{c}_t^* \equiv \log\left(\frac{C_t^*}{C^*}\right)$ Eq.(3-9) to derive Eq.(3-10). Eq.(3-10) is identical with Eq.(24) in the text.

3.3 Log-linearizing Euler Equation

Total derivative of Eq.(1-6) is given by:

$$dU_{c,t} = U_c\beta d(1+i_t) + U_c(-1)d\Pi_{t+1} + dU_{c,t+1} - U_c dZ_t + U_c dZ_{t+1}.$$

Note that $\beta = (1+i)^{-1}$ and $i = \rho$. Thus:

$$dU_{c,t} = U_c\beta\frac{d(1+i_t)}{1+\rho} + U_c(-1)d\Pi_{t+1} + dU_{c,t+1} - U_c dZ_t + U_c dZ_{t+1}.$$

Dividing both sides of the previous expression by U_c yields:

$$\frac{dU_{c,t}}{U_c} = \frac{d(1+i_t)}{1+\rho} - d\Pi_{t+1} + \frac{dU_{c,t+1}}{U_c} - \left[-\left(dZ_{t+1} - dZ_t\right)\right],$$

which can be rewritten as:

$$\log\left(\frac{U_{c,t}}{U_c}\right) = \log\left(\frac{1+i_t}{1+\rho}\right) - \log\Pi_{t+1} + \log\left(\frac{U_{c,t+1}}{U_c}\right) - \left[-\log\left(\frac{Z_{t+1}}{Z_t}\right)\right].$$

Let define $\hat{i}_t \equiv \log\left(\frac{1+i_t}{1+\rho}\right)$, $\pi_t \equiv \log\Pi_t$ and $\hat{\rho}_t \equiv -\log\left(\frac{Z_{t+1}}{Z_t}\right)$. Then, the previous expression can be rewritten as:

$\hat{\xi}_t = \hat{\xi}_{t+1} + \hat{i}_t - \pi_{t+1} - \hat{\rho}_t$, (3-11)

which is a class of log-linearized Euler equation. Eq. (3-11) is identical with Eq(6-5) in the text.

3.4 Log-linearizing Marginal Utility of Consumption

Marginal utility of consumption can be depicted as:

$U_{c,t} = U_{l,t}\left(\frac{U_{l,t}}{U_{c,t}}\right)^{-1}$. (3-12)

Total derivative of Eq.(3-12) is given by:

$$
\begin{aligned}
dU_{c,t} &= \left\{ U_{ll}\left(\frac{U_l}{U_c}\right)^{-1} + U_l(-1)\left(\frac{U_l}{U_c}\right)^{-2}\left[\frac{1}{U_c}U_{ll} + U_l(-1)U_c^{-2}\frac{\partial U_c}{\partial L}\right]\right\} dL_t \\
&\quad + \left\{ U_{lc}\left(\frac{U_l}{U_c}\right)^{-1} + U_l(-1)\left(\frac{U_l}{U_c}\right)^{-2}\left[\frac{1}{U_c}U_{lc} + U_l(-1)U_c^{-2}\frac{\partial U_c}{\partial C}\right]\right\} dC_t \\
&= U_l^2\left(\frac{U_c}{U_l}\right)^2 \frac{1}{U_c^2}U_{cl}L\frac{dL_t}{L} + \left[U_{lc}\frac{U_c}{U_l}C - U_l\left(\frac{U_c}{U_l}\right)^2\left(\frac{U_{lc}}{U_c}C - \frac{U_l}{U_c}\frac{U_{cc}}{U_c}C\right)\right]\frac{dC_t}{C} \\
&= U_{cl}L\frac{dL_t}{L} + \left[\frac{U_{lc}}{U_l}U_cC - U_c\left(\frac{U_{lc}}{U_l}C - \frac{U_{cc}}{U_c}C\right)\right]\frac{dC_t}{C}
\end{aligned}.
$$

Dividing both sides of the previous expression by U_c yields:

$$
\begin{aligned}
\frac{dU_{c,t}}{U_c} &= \frac{U_{cl}}{U_c}L\frac{dL_t}{L} + \left(\frac{U_{lc}}{U_l}C - \frac{U_{lc}}{U_l}C + \frac{U_{cc}}{U_c}C\right)\frac{dC_t}{C} \\
&= \frac{U_{cl}}{U_c}L\frac{dL_t}{L} + \frac{U_{cc}}{U_c}C\frac{dC_t}{C}
\end{aligned},
$$

which can be rewritten as:

$$\hat{\xi}_t = \upsilon\hat{l}_t - \sigma\hat{c}_t,$$

which is identical with Eq(26) in the text.

3.5 Deriving the FONC for Domestic Producers

Total derivative of Eq.(1-23) yields:

$$
\begin{aligned}
d\tilde{X}_{H,t} &= \frac{\varepsilon}{\varepsilon-1}\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]^{-1}\left[\begin{aligned}&dMC_{t|t}+\theta\beta dMC_{t+1|t}\\&+(\theta\beta)^2 dMC_{t+2|t}+\cdots\end{aligned}\right] \\
&\quad + \frac{\varepsilon}{\varepsilon-1}(MC)\left[d\Pi_{H,t}+\theta\beta d\Pi_{H,t+1}+(\theta\beta)^2 d\Pi_{H,t+2}+\cdots\right]
\end{aligned}
$$

Note that $1+\theta\beta+(\theta\beta)^2+\cdots=\frac{1}{1-\theta\beta}$ and $MC=\frac{\varepsilon-1}{\varepsilon}$. Then, the previous expression can be rewritten as:

$$
\begin{aligned}
d\tilde{X}_{H,t} &= (1-\theta\beta)\left[\frac{dMC_{t|t}}{MC}+\theta\beta\frac{dMC_{t+1|t}}{MC}+(\theta\beta)^2\frac{dMC_{t+2|t}}{MC}+\cdots\right] \\
&\quad + \left[d\Pi_{H,t}+\theta\beta d\mathrm{E}_t\left(\Pi_{H,t+1}\right)+(\theta\beta)^2 d\mathrm{E}_t\left(\Pi_{H,t+2}\right)+\cdots\right]
\end{aligned},
$$

which can be rewritten as:

$$\tilde{p}_{H,t} - p_{H,t-1} = (1-\theta\beta)\left[\widehat{mc}_{t|t} + \theta\beta\widehat{mc}_{t+1|t} + (\theta\beta)^2 \widehat{mc}_{t+2|t} + \cdots\right], \\ + \left[\pi_{H,t} + \theta\beta\pi_{H,t+1} + (\theta\beta)^2 \pi_{H,t+2} + \cdots\right]$$

with $\widehat{mc}_{t+k|t} \equiv \log\left(\frac{MC_{t+k|t}}{MC}\right) = mc_{t+k|t} - mc$, $mc_t \equiv \log MC_t$ and $mc \equiv \log MC = -\log\left(\frac{\varepsilon-1}{\varepsilon}\right)$.

Previous expression can be rewritten as:

$$\begin{aligned} \tilde{p}_{H,t} - p_{H,t-1} &= (1-\theta\beta)\left[\widehat{mc}_{t|t} + \theta\beta\widehat{mc}_{t+1|t} + (\theta\beta)^2 \widehat{mc}_{t+2|t} + \cdots\right] \\ &\quad + \left[p_{H,t} - p_{H,t-1} + \theta\beta(p_{H,t+1} - p_{H,t}) + (\theta\beta)^2 (p_{H,t+2} - p_{H,t+1}) + \cdots\right] \\ &= (1-\theta\beta)\left[\widehat{mc}_{t|t} + \theta\beta\widehat{mc}_{t+1|t} + (\theta\beta)^2 \widehat{mc}_{t+2|t} + \cdots\right] \\ &\quad + \left[(1-\theta\beta)p_{H,t} + \theta\beta(1-\theta\beta)p_{H,t+1} + (\theta\beta)^2(1-\theta\beta)p_{H,t+2} + \cdots\right] - p_{H,t-1} \\ &= (1-\theta\beta)\left(\widehat{mc}_{t|t} + p_{H,t}\right) + \theta\beta(1-\theta\beta)\left(\widehat{mc}_{t+1|t} + p_{H,t+1}\right) \\ &\quad + (\theta\beta)^2(1-\theta\beta)\left(\widehat{mc}_{t+2|t} + p_{H,t+2}\right) + \cdots - p_{H,t-1} \end{aligned}.$$

(3-13)

Note that:

$$\begin{aligned} \widehat{mc}_{t+k|t} + p_{H,t+k} &= mc_{t+k|t} + p_{H,t+k} - \log MC \\ &= mc^n_{t+k|t} - \log MC \\ &= mc^n_{t+k|t} - \log\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-1} \\ &= mc^n_{t+k|t} + \log\left(\frac{\varepsilon}{\varepsilon-1}\right) \\ &= mc^n_{t+k|t} + \mu \end{aligned} \quad . \text{ (3-14)}$$

Plugging Eq.(3-14) into Eq.(3-13) yields:

$$\begin{aligned} \tilde{p}_{H,t} - p_{H,t-1} &= (1-\theta\beta)\left(mc^n_{t|t} + \mu\right) + \theta\beta(1-\theta\beta)\left(mc^n_{t+1|t} + \mu\right) \\ &\quad + (\theta\beta)^2(1-\theta\beta)\left(mc^n_{t+2|t} + \mu\right) + \cdots - p_{H,t-1} \\ &= (1-\theta\beta)\left[mc^n_{t|t} + \theta\beta mc^n_{t+1|t} + (\theta\beta)^2 mc^n_{t+2|t} + \cdots\right] - p_{H,t-1} \\ &\quad + (1-\theta\beta)\left[1 + \theta\beta + (\theta\beta)^2 + \cdots\right]\mu \\ &= \mu + (1-\theta\beta)\left[mc^n_{t|t} + \theta\beta mc^n_{t+1|t} + (\theta\beta)^2 mc^n_{t+2|t} + \cdots\right] - p_{H,t-1} \end{aligned},$$

which can be rewritten as:

$$\tilde{p}_{H,t} = \mu + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k mc_{t+k|t}^{n} \text{. (3-15)}$$

(Corresponding to Eq.11 in Chap. 3, Gali, 2015)

Eq.(3-13) can be rewritten as:

$$\begin{aligned}\tilde{p}_{H,t} - p_{H,t-1} &= (1-\theta\beta)\left[\widehat{mc}_{t|t} + \theta\beta\widehat{mc}_{t+1|t} + (\theta\beta)^2\widehat{mc}_{t+2|t} + \cdots\right] \\ &\quad + \left[p_{H,t} - p_{H,t-1} + \theta\beta\left(p_{H,t+1} - p_{H,t}\right) + (\theta\beta)^2\left(p_{H,t+2} - p_{H,t+1}\right) + \cdots\right] \\ &= (1-\theta\beta)\left[\widehat{mc}_{t|t} + \theta\beta\widehat{mc}_{t+1|t} + (\theta\beta)^2\widehat{mc}_{t+2|t} + \cdots\right] \\ &\quad + \left[(1-\theta\beta)p_{H,t} + \theta\beta(1-\theta\beta)p_{H,t+1} + (\theta\beta)^2(1-\theta\beta)p_{H,t+2} + \cdots\right] - p_{H,t-1} \\ &= (1-\theta\beta)\left(\widehat{mc}_{t|t} + p_{H,t}\right) + \theta\beta(1-\theta\beta)\left(\widehat{mc}_{t+1|t} + p_{H,t+1}\right) \\ &\quad + (\theta\beta)^2(1-\theta\beta)\left(\widehat{mc}_{t+2|t} + p_{H,t+2}\right) + \cdots - p_{H,t-1}\end{aligned}$$

Eq.(3-13) can be rewritten as:

$$\tilde{p}_{H,t} - p_{H,t-1} = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\widehat{mc}_{t+k|t} + \sum_{k=0}^{\infty}(\theta\beta)^k\pi_{H,t+k} \text{. (3-16)}$$

Eq.(3-16) can be rewritten as:

$$\tilde{p}_{H,t} - p_{H,t-1} = (1-\theta\beta)\widehat{mc}_{t|t} + \pi_{H,t} + (1-\theta\beta)\sum_{k=1}^{\infty}(\theta\beta)^k\widehat{mc}_{t+k|t} + \sum_{k=1}^{\infty}(\theta\beta)^k\pi_{H,t+k} \text{. (3-17)}$$

Forwarding Eq.(3-17) one period yields:

$$\tilde{p}_{H,t+1} - p_{H,t} = \frac{1-\theta\beta}{\theta\beta}\sum_{k=1}^{\infty}(\theta\beta)^k\widehat{mc}_{t+k|t} + \frac{1}{\theta\beta}\sum_{k=1}^{\infty}(\theta\beta)^k\pi_{H,t+k}.$$

Multiplying $\theta\beta$ on the both sides of the previous expression yields:

$$\theta\beta\left(\tilde{p}_{H,t+1} - p_{H,t}\right) = (1-\theta\beta)\sum_{k=1}^{\infty}(\theta\beta)^k\widehat{mc}_{t+k|t} + \sum_{k=1}^{\infty}(\theta\beta)^k\pi_{H,t+k}.$$

Plugging the previous expression into Eq.(3-17) yields:

$$\tilde{p}_{H,t} - p_{H,t-1} = \theta\beta\left(\tilde{p}_{H,t+1} - p_{H,t}\right) + (1-\theta\beta)\widehat{mc}_{t|t} + \pi_{H,t} \text{. (3-18)}$$

Calvo-pricing’s transitory equation is given by:

$$P_{H,t} = \left[\theta P_{H,t-1}^{1-\varepsilon} + (1-\theta)\tilde{P}_{H,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Log-linearizing the previous expression around the steady state yields:

$$p_{H,t} = \theta p_{H,t-1} + (1-\theta)\tilde{p}_{H,t}.$$

Subtracting $p_{H,t-1}$ from the both sides of the previous expression yields:

$\pi_{H,t} = (1-\theta)(\tilde{p}_{H,t} - p_{H,t-1})$. (3-19)

Plugging Eq.(3-19) into Eq.(3-18) yields:

$$\frac{1}{1-\theta}\pi_{H,t} = \theta\beta\frac{1}{1-\theta}\pi_{H,t+1} + (1-\theta\beta)\widehat{mc}_{t|t} + \pi_{H,t},$$

which can be rewritten as:

$$\frac{\theta}{1-\theta}\pi_{H,t} = \theta\beta\frac{1}{1-\theta}\pi_{H,t+1} + (1-\theta\beta)\widehat{mc}_{t|t}.$$

Multiplying both sides of the previous expression by $\frac{1-\theta}{\theta}$ yields:

$$\pi_{H,t} = \beta\pi_{H,t+1} + \frac{(1-\theta\beta)(1-\theta)}{\theta}\widehat{mc}_{t|t}.$$

Let assume $Y_{t+k} = N_{t+k|t}^{1-\alpha}\Omega_{t+k}^{-(1-\alpha)}$. Then, the (nominal) marginal cost for an individual firm that last set its price is given by:

$$\begin{aligned} MC_{t+k|t}^{n} &= W_{t+k}\frac{\partial N_{t+k|t}}{\partial Y_{t+k}} \\ &= W_{t+k} MPN_{t+k|t}^{-1} \\ &= W_{t+k}\left(\frac{\partial Y_{t+k}}{\partial N_{t+k|t}}\right)^{-1} \\ &= W_{t+k}\left(\frac{\partial N_{t+k|t}^{1-\alpha}\Omega_{t+k}^{-(1-\alpha)}}{\partial N_{t+k|t}}\right)^{-1} \\ &= W_{t+k}\left[(1-\alpha)N_{t+k|t}^{-\alpha}\right]^{-1}\Omega_{t+k}^{1-\alpha} \end{aligned}.$$

Note that the (nominal) average marginal cost is given by:

$$\begin{aligned} MC_{t+k}^{n} &= W_{t+k}\frac{\partial N_{t+k}}{\partial Y_{t+k}} \\ &= W_{t+k}\left(\frac{\partial Y_{t+k}}{\partial N_{t+k}}\right)^{-1} \\ &= W_{t+k}\left[(1-\alpha)N_{t+k}^{-\alpha}\right]^{-1}\Omega_{t+k}^{1-\alpha} \end{aligned}$$

Total derivative of the (nominal) marginal cost for an individual firm that last set its price is given by:

$$
\begin{aligned}
dMC^n_{t+k|t} &= \frac{N^\alpha}{1-\alpha} dW_{t+k} + W(-1)\left[(1-\alpha)N^{-\alpha}\right]^{-2}(1-\alpha)(-\alpha)N^{-\alpha-1}dN_{t+k|t} \\
&= \frac{WN^\alpha}{1-\alpha}\frac{dW_{t+k}}{W} + W\frac{1}{(1-\alpha)N^{-\alpha}}\frac{(1-\alpha)\alpha N^{-\alpha}}{(1-\alpha)N^{-\alpha}}\frac{dN_{t+k|t}}{N} \\
&= \frac{WN^\alpha}{1-\alpha}\frac{dW_{t+k}}{W} + \frac{WN^\alpha}{1-\alpha}\alpha\frac{dN_{t+k|t}}{N}
\end{aligned}
.
$$

Dividing both sides of the previous expression by MC^n yields:

$$
\begin{aligned}
\frac{dMC^n_{t+k|t}}{MC^n} &= \frac{1-\alpha}{WN^\alpha}\left(\frac{WN^\alpha}{1-\alpha}\frac{dW_{t+k}}{W} + \frac{WN^\alpha}{1-\alpha}\alpha\frac{dN_{t+k|t}}{N}\right) \\
&= \frac{dW_{t+k}}{W} + \alpha\frac{dN_{t+k|t}}{N}
\end{aligned}
,
$$

which can be rewritten as:

$mc^n_{t+k|t} = w_{t+k} + \alpha\hat{n}_{t+k|t}$. (3-20)

Log-linearization of the average (log) marginal cost is given by:

$mc^n_{t+k} = w_{t+k} + \alpha\hat{n}_{t+k}$. (3-21)

Subtracting Eq.(3-21) from Eq.(3-20) yields:

$mc^n_{t+k|t} - mc^n_{t+k} = \alpha\left(\hat{n}_{t+k|t} - \hat{n}_{t+k}\right)$,

which can be rewritten as:

$mc^n_{t+k|t} = mc^n_{t+k} + \alpha\left(\hat{n}_{t+k|t} - \hat{n}_{t+k}\right)$.

Plugging the logarithmic production function $\hat{y}_{t+k|t} = (1-\alpha)\hat{n}_{t+k|t}$ and $\hat{y}_t = (1-\alpha)\hat{n}_t$ into the previous expression yields:

$mc^n_{t+k|t} = mc^n_{t+k} + \frac{\alpha}{1-\alpha}\left(\hat{y}_{t+k|t} - \hat{y}_{t+k}\right)$.

Plugging logarithmic demand function of $Y_{t+k|t} \equiv \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} Y_{t+k}$ which is given by $\hat{y}_{t+k|t} = -\varepsilon\left(\tilde{p}_{H,t} - p_{H,t+k}\right) + \hat{y}_{t+k}$ into the previous expression yields:

$mc^n_{t+k|t} = mc^n_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\left(\tilde{p}_{H,t} - p_{H,t+k}\right)$. (3-22)

Plugging Eq.(3-22) into Eq.(3-15) yields:

$$\tilde{p}_{H,t} = \mu + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \left[mc_{t+k}^n - \frac{\alpha\varepsilon}{1-\alpha}\left(\tilde{p}_{H,t} - p_{H,t+k}\right)\right] \quad ,$$
$$= \mu + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \left(mc_{t+k}^n + \frac{\alpha\varepsilon}{1-\alpha} p_{H,t+k}\right) - \frac{\alpha\varepsilon}{1-\alpha}\tilde{p}_{H,t}$$

which can be rewritten as:

$$\frac{(1-\alpha)+\alpha\varepsilon}{1-\alpha}\tilde{p}_{H,t} = \mu + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \left(mc_{t+k}^n + \frac{\alpha\varepsilon}{1-\alpha} p_{H,t+k}\right)$$
$$= \mu + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \left(p_{H,t+k} - \mu_{t+k} + \frac{\alpha\varepsilon}{1-\alpha} p_{H,t+k}\right) ,$$
$$= \mu + (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \left(-\mu_{t+k} + \frac{(1-\alpha)+\alpha\varepsilon}{1-\alpha} p_{H,t+k}\right)$$
$$= (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \left(\mu - \mu_{t+k} + \frac{(1-\alpha)+\alpha\varepsilon}{1-\alpha} p_{H,t+k}\right)$$

where we use the definition of the (log) desired markup $\mu_t \equiv -\left(mc_t^n - p_{H,t}\right)$ which is (log) inverse of the real marginal cost.

Let define $\hat{\mu}_t \equiv \mu_t - \mu$ being the deviation between the average and desired marginal cost. Plugging the definition into the previous expression yields:

$$\tilde{p}_{H,t} = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k \left[p_{H,t+k} - \frac{1-\alpha}{(1-\alpha)+\alpha\varepsilon}\hat{\mu}_{t+k}\right]. \quad (3\text{-}23)$$

with $\Theta \equiv \frac{1-\alpha}{(1-\alpha)+\alpha\varepsilon}$.

Eq.(3-23) can be rewritten as:

$$
\begin{aligned}
\tilde{p}_{H,t}-p_{H,t-1} &= (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\left(p_{H,t+k}-\Theta\hat{\mu}_{t+k}\right)-p_{H,t-1} \\
&= (1-\theta\beta)\left[p_{H,t}+\theta\beta p_{H,t+1}+(\theta\beta)^2 p_{H,t+2}+\cdots\right]-p_{H,t-1}-(1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\Theta\hat{\mu}_{t+k} \\
&= -p_{H,t-1}+\left[p_{H,t}+\theta\beta p_{H,t+1}+(\theta\beta)^2 p_{H,t+2}+\cdots\right]-\theta\beta\left[p_{H,t}+\theta\beta p_{H,t+1}+(\theta\beta)^2 p_{H,t+2}+\cdots\right] \\
&\quad -(1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\Theta\hat{\mu}_{t+k} \\
&= p_{H,t}-p_{H,t-1}+\theta\beta\left(p_{H,t+1}-p_{H,t}\right)+(\theta\beta)^2\left(p_{H,t+2}-p_{H,t+1}\right)+(\theta\beta)^3\left(p_{H,t+3}-p_{H,t+2}\right)+\cdots \\
&\quad -(1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\Theta\hat{\mu}_{t+k} \\
&= \pi_{H,t}+\theta\beta\pi_{H,t+1}+(\theta\beta)^2\pi_{H,t+2}+(\theta\beta)^3\pi_{H,t+3}+\cdots-(1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\Theta\hat{\mu}_{t+k} \\
&= \sum_{k=0}^{\infty}(\theta\beta)^k\pi_{H,t+k}-(1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\Theta\hat{\mu}_{t+k}
\end{aligned}
$$

, (3-24)

Forwarding Eq.(3-24) one period yields:

$$
\begin{aligned}
\tilde{p}_{H,t+1}-p_{H,t} &= \pi_{H,t+1}+\theta\beta\pi_{H,t+2}+(\theta\beta)^2\pi_{H,t+3}+(\theta\beta)^3\pi_{H,t+4}+\cdots \\
&\quad -(1-\theta\beta)\Theta\left\{\hat{\mu}_{t+1}+\theta\beta\hat{\mu}_{t+2}+(\theta\beta)^2\hat{\mu}_{t+3}+(\theta\beta)^3\hat{\mu}_{t+4}+\cdots\right\} \\
&= \frac{1}{\theta\beta}\sum_{k=1}^{\infty}(\theta\beta)^k\pi_{H,t+k}-\frac{(1-\theta\beta)\Theta}{\theta\beta}\sum_{k=1}^{\infty}(\theta\beta)^k\hat{\mu}_{t+k}
\end{aligned}
$$

Multiplying $\theta\beta$ on the both sides of the previous expression yields:

$$
\theta\beta\left(\tilde{p}_{H,t+1}-p_{H,t}\right)=\sum_{k=1}^{\infty}(\theta\beta)^k\pi_{H,t+k}-(1-\theta\beta)\sum_{k=1}^{\infty}(\theta\beta)^k\Theta\hat{\mu}_{t+k}.
$$

Plugging the previous expression into Eq.(3-24) yields:

$$
\begin{aligned}
\tilde{p}_{H,t}-p_{H,t-1} &= \pi_{H,t}+\theta\beta\pi_{H,t+1}+(\theta\beta)^2\pi_{H,t+2}+(\theta\beta)^3\pi_{H,t+3}+\cdots \\
&\quad -(1-\theta\beta)\left[\Theta\hat{\mu}_t+\theta\beta\Theta\hat{\mu}_{t+1}+(\theta\beta)^2\hat{\mu}_{t+2}+(\theta\beta)^3\hat{\mu}_{t+3}\right] \\
&= \pi_{H,t}-(1-\theta\beta)\Theta\hat{\mu}_t+\sum_{k=1}^{\infty}(\theta\beta)^k\pi_{H,t+k}-(1-\theta\beta)\sum_{k=1}^{\infty}(\theta\beta)^k\Theta\hat{\mu}_{t+k} \\
&= \pi_{H,t}-(1-\theta\beta)\Theta\hat{\mu}_t+\theta\beta\left(\tilde{p}_{H,t+1}-p_{H,t}\right)
\end{aligned}.
$$

Plugging Eq.(3-19) into the previous expression yields:

$$
\frac{1}{1-\theta}\pi_{H,t}=\pi_{H,t}-(1-\theta\beta)\Theta\hat{\mu}_t+\theta\beta\frac{1}{1-\theta}\pi_{H,t+1},
$$

which can be rewritten as:

$$\pi_{H,t} = \frac{1-\theta}{\theta}\left[-(1-\theta\beta)\Theta\hat{\mu}_t + \theta\beta\frac{1}{1-\theta}\pi_{H,t+1}\right].$$

Then, we have:

$\pi_{H,t} = \beta\pi_{H,t+1} - \kappa\hat{\mu}_t$, (3-25)

with $\kappa \equiv \frac{(1-\theta\beta)(1-\theta)}{\theta}\Theta$. Eq.(3-25) is identical with Eq.(27) in the text.

3.6 Log-linearization of Intra-temporal Optimality Condition

Dividing both sides of Eq.(1-7) by $P_{H,t}/P_{H,t}$ yields:

$$\frac{W_t}{P_{H,t}} = \frac{V_{n,t}}{U_{c,t}}S_t^{\nu}.$$

Plugging the previous expression into Eq.(1-24) yields:

$$MC_t = \frac{V_{n,t}}{U_{c,t}}S_t^{\nu}\frac{N_t^{\alpha}}{(1-\alpha)}.$$

Total derivative of the previous expression is given by:

$$\begin{aligned} dMC_t &= \left(\frac{1}{U_c}\frac{N^{\alpha}}{1-\alpha}\frac{\partial V_n}{\partial N} + \frac{V_n}{U_c}\frac{\alpha}{1-\alpha}\frac{N^{\alpha}}{N}\right)dN_t - U_c^{-2}V_n\frac{N^{\alpha}}{1-\alpha}dU_{c,t} + \frac{V_n}{U_c}\frac{N^{\alpha}}{1-\alpha}\nu dS_t \\ &= \frac{V_n}{U_c}\frac{N^{\alpha}}{1-\alpha}\left(\frac{V_{nn}N}{V_n}+\alpha\right)\frac{dN_t}{N} - \frac{V_n}{U_c}\frac{N^{\alpha}}{1-\alpha}\frac{dU_{c,t}}{U_c} + \frac{V_n}{U_c}\frac{N^{\alpha}}{1-\alpha}\nu dS_t \end{aligned}.$$

Plugging Eq.(2-9) into the previous expression yields:

$$\begin{aligned} dMC_t &= \frac{1-\alpha}{N^{\alpha}\mu S^{\nu}}\frac{N^{\alpha}}{1-\alpha}\left(\frac{V_{nn}N}{V_n}+\alpha\right)\frac{dN_t}{N} - \frac{1-\alpha}{N^{\alpha}\mu S^{\nu}}\frac{N^{\alpha}}{1-\alpha}\frac{dU_{c,t}}{U_c} \\ &\quad + \frac{1-\alpha}{N^{\alpha}\mu S^{\nu}}\frac{N^{\alpha}}{1-\alpha}\nu dS_t \\ &= \mu^{-1}\left(\frac{V_{nn}N}{V_n}+\alpha\right)\frac{dN_t}{N} - \mu^{-1}\frac{dU_{c,t}}{U_c} + \mu^{-1}\nu dS_t \end{aligned}.$$

Multiplying both sides of the previous expression by μ yields:

$$\frac{dMC_t}{MC} = \left(\frac{V_{nn}N}{V_n}+\alpha\right)\frac{dN_t}{N} - \frac{dU_{c,t}}{U_c} + \nu dS_t,$$

which can be rewritten as:

$$\log\left(\frac{MC_t}{MC}\right)=\left(\frac{V_{nn}N}{V_n}+\alpha\right)\log\left(\frac{N_t}{N}\right)-\log\left(\frac{U_{c,t}}{U_c}\right)+\nu\log S_t .$$

By using the definition of $\widehat{mc}_t\equiv\log\left(\frac{MC_t}{MC}\right)$, $\hat{n}_t\equiv\log\left(\frac{N_t}{N}\right)$, $\hat{\xi}_t\equiv\log\left(\frac{U_{c,t}}{U_c}\right)$ and $\varphi\equiv\frac{V_{nn}N}{V_n}$, the previous expression can be rewritten as:

$\widehat{mc}_t=(\varphi+\alpha)\hat{n}_t-\hat{\xi}_t+\nu s_t$, (3-26)

where we use Eq.(3-9).

Eq.(3-14) implies as follows:

$$\begin{aligned}\widehat{mc}_t+p_{H,t}&=mc_t^n+\mu\\&=-\mu_t+p_{H,t}+\mu,\\&=-\hat{\mu}_t+p_{H,t}\end{aligned}$$

or $\widehat{mc}_t=-\hat{\mu}_t$,

where we use $mc_t^n=-\mu_t+p_{H,t}$ which is derived by the definition of the desired markup. Plugging the previous expression into Eq.(3-26) yields:

$$\hat{\mu}_t=\hat{\xi}_t-(\varphi+\alpha)\hat{n}_t-\nu s_t .$$

Plugging the (log) production function $\hat{y}_t=(1-\alpha)\hat{n}_t$ derived from Eq.(1-21) into the previous expression yields:

$$\hat{\mu}_t=\hat{\xi}_t-\frac{\varphi+\alpha}{1-\alpha}\hat{y}_t-\nu s_t ,$$

which is identical with Eq.(28) in the text.

3.7 Deriving the LM Equation

Eq.(1-8) can be rewritten as:

$$\frac{U_{l,t}}{U_{c,t}}=\frac{i_t}{1+i_t} .$$

Multiplying —1 on both sides of the previous expression yields:

$$-\frac{U_{l,t}}{U_{c,t}}=-\frac{i_t}{1+i_t}$$

Summing 1 both sides of the previous expression yields:

$$
\begin{aligned}
1-\frac{U_{l,t}}{U_{c,t}} &= 1-\frac{i_t}{1+i_t} \\
&= \frac{1+i_t}{1+i_t}-\frac{i_t}{1+i_t}. \\
&= \frac{1}{1+i_t}
\end{aligned}
$$

Raise both sides of the previous expression to power －1 yields:

$$
1+i_t=\left(1-\frac{U_{l,t}}{U_{c,t}}\right)^{-1}.
$$

Total derivative of the previous expression yields:

$$
\begin{aligned}
d\left(1+i_t\right) &= -\left(1-\frac{U_l}{U_c}\right)^{-2}\left[-U_l\left(-1\right)U_c^{-2}U_{cc}-\frac{1}{U_c}U_{lc}\right]dC_t+\left[-\frac{1}{U_c}U_{ll}-U_l\left(-1\right)U_c^{-2}U_{cl}\right]dL_t \\
&= -\left(1+\rho\right)^{-2}\left[\left(\frac{U_l}{U_c}\frac{U_{cc}}{U_c}-\frac{U_{lc}}{U_c}\right)dC_t+\left(-\frac{U_l}{U_c}\frac{U_{ll}}{U_l}+\frac{U_l}{U_c}\frac{U_{lc}}{U_c}\right)dL_t\right] \\
&= -\left(1+\rho\right)^{-2}\frac{U_l}{U_c}\left[\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)\frac{dC_t}{C}+\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\frac{dL_t}{L}\right] \\
&= -\left(1+\rho\right)\rho\left[\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)\frac{dC_t}{C}+\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\frac{dL_t}{L}\right]
\end{aligned}
.
$$

Dividing both sides of the previous expression by $1+\rho$ yields:

$$
\begin{aligned}
\frac{d\left(1+i_t\right)}{1+\rho} &= -\rho\left[\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)\frac{dC_t}{C}+\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\frac{dL_t}{L}\right] \\
&= -\rho\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)\frac{dC_t}{C}-\rho\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\frac{dL_t}{L}
\end{aligned}
,
$$

which can be rewritten as:

$$
\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\frac{dL_t}{L}=-\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)\frac{dC_t}{C}-\frac{1}{\rho}\frac{d\left(1+i_t\right)}{1+\rho}. \text{ (3-27)}
$$

Iff $-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L-\left[-\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)\right]=0$, (3-28)

$$-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L=-\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right), \text{(3-29)}$$

is applicable.

Let assume $U(C,L)=\frac{1}{1-\nu}\left(C^{1-\vartheta}L^{\vartheta}\right)^{1-\nu}$. Then, we have:

$$U_c=(1-\vartheta)h'\left(\frac{L}{C}\right)^{\vartheta},$$

$$U_{cc}=(1-\vartheta)\left[-\sigma(1-\vartheta)-\vartheta\right]h'\left(\frac{L}{C}\right)^{\vartheta}C^{-1},$$

$$U_{cl}=\vartheta(1-\vartheta)(1-\nu)h'\left(\frac{L}{C}\right)^{\vartheta}L^{-1}, \qquad \text{(3-30)}$$

$$U_l=\vartheta h'\left(\frac{L}{C}\right)^{\vartheta-1},$$

$$U_{ll}=\vartheta\left[-\sigma\vartheta-(1-\vartheta)\right]h'\left(\frac{L}{C}\right)^{\vartheta-1}L^{-1},$$

with $h'\equiv\left(C^{1-\vartheta}L^{\vartheta}\right)^{-\nu}$.

Plugging Eq.(3-30) into the RHS of Eq.(3-29) yields:

$$-\left(\frac{U_{cc}}{U_c}C-\frac{U_c}{U_l}\frac{U_{lc}}{U_c}C\right)=-\left[-\sigma(1-\vartheta)-\vartheta-(1-\vartheta)(1-\nu)\right]. \text{(3-31)}$$
$$=1$$

Plugging Eq.(3-30) into the LHS of Eq.(3-29) yields:

$$-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L=-\left[-\nu\vartheta-(1-\vartheta)\right]+\vartheta(1-\nu). \text{(3-32)}$$
$$=1$$

Plugging Eqs.(3-31) and (3-32) into the LHS of Eq.(3-28) yields:

1—1＝0.

Thus, Eq.(3-29) is applicable. Plugging Eq.(3-29) into Eq.(3-27) yields:

$$\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\log\left(\frac{L_t}{L}\right)=-\left(-\frac{U_{ll}}{U_l}L+\frac{U_{lc}}{U_c}L\right)\log\left(\frac{C_t}{C}\right)-\frac{1}{\rho}\log\left(\frac{1+i_t}{1+\rho}\right),$$

where we use the fact that $\frac{dL_t}{L}=\log\left(\frac{L_t}{L}\right)$, $\log\left(\frac{C_t}{C}\right)$ and $\frac{d(1+i_t)}{1+\rho}=\log\left(\frac{1+i_t}{1+\rho}\right)$.

By using definitions $\sigma_l \equiv -\frac{U_{ll}}{U_l}L$ and $\upsilon \equiv \frac{U_{lc}}{U_c}L$, the previous expression can be rewritten as:

$$(\sigma_l + \upsilon)\hat{l}_t = (\sigma_l + \upsilon)\hat{c}_t - \frac{1}{\rho}\hat{i}_t,$$

which can be rewritten as:

$$\hat{l}_t = \hat{c}_t - \frac{1}{\rho(\sigma_l + \upsilon)}\hat{i}_t.$$

By using the definition $\varepsilon_{lc} \equiv \frac{1}{\sigma_l + \upsilon}$, the previous expression can be rewritten as:

$$\hat{l}_t = \hat{c}_t - \eta\hat{i}_t, \text{ (3-33)}$$

with $\eta \equiv \frac{\varepsilon_{lc}}{\rho}$. Eq.(3-33) is identical with Eq(29) in the text.

3.8 Relationship between Changes in the Real Money Balance and Inflation

Total derivative of the definition of the real money balance $L_t \equiv \frac{M_t}{P_t}$ is given by:

$$\begin{aligned} dL_t &= \frac{dM_t}{P} + (-1)P^{-2}MdP_t \\ &= \frac{M}{P}\frac{dM_t}{M} - \frac{M}{P}\frac{dP}{M} \\ &= L\frac{dM_t}{M} - L\frac{dP}{M} \end{aligned}.$$

Dividing both sides of the previous expression yields:

$$\hat{l}_t = \log\left(\frac{M_t}{M}\right) - \log\left(\frac{P_t}{M}\right). \text{ (3-34)}$$

First differential equation of Eq.(3-34) is given by:

$$\begin{aligned} \hat{l}_t - \hat{l}_{t-1} &= \log\left(\frac{M_t}{M}\right) - \log\left(\frac{M_{t-1}}{M}\right) - \left[\log\left(\frac{P_t}{M}\right) - \log\left(\frac{P_{t-1}}{M}\right)\right] \\ &= \log M_t - \log M_{t-1} - \left(\log P_t - \log P_{t-1}\right) \\ &= -\pi_t + \Delta m_t \end{aligned},$$

which can be rewritten as:

$\hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t$. (3-35)

Eq.(3-35) is identical with Eq.(30) in the text.

3.9 Log-linearization of the Consolidated Government Budget Constraint

By using the definition $g(S_t) \equiv \frac{P_t}{P_{H,t}} = S_t^{\nu}$ and $\mathrm{P}_t \equiv (1+i_t)\frac{P_t}{P_{t+1}}$, Eq(3-1) can be rewritten as:

$$g(S_t)^{-1} G_t + \mathrm{B}_{t-1}(1+i_{t-1})\Pi_t^{-1} = T_t + \mathrm{B}_t + \frac{\Delta M_t}{P_t}$$

with $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. The previous expression can be rewritten as:

$$\mathrm{B}_t = S_t^{-\nu} G_t + \mathrm{B}_{t-1}(1+i_{t-1})\Pi_t^{-1} - T_t - \frac{\Delta M_t}{P_t} .$$

Total derivative of the previous expression yields:

$$\begin{aligned} d\mathrm{B}_t &= G(-\nu)dS_t + dG_t + (1+\rho)d\mathrm{B}_{t-1} + \mathrm{B}d(1+i_{t-1}) + \mathrm{B}(1+\rho)(-1)d\Pi_t \\ &\quad -dT_t - d(\Delta M_t/P_t) \end{aligned} .$$

Dividing both sides of the previous expression by Y yields:

$$\begin{aligned} \frac{d\mathrm{B}_t}{Y} &= -\frac{G}{Y}\nu dS_t + \frac{dG_t}{Y} + (1+\rho)\frac{d\mathrm{B}_{t-1}}{Y} + \frac{\mathrm{B}}{Y}d(1+i_{t-1}) - \frac{\mathrm{B}}{Y}(1+\rho)d\Pi_t \\ &\quad -\frac{dT_t}{Y} - \frac{d(\Delta M_t/P_t)}{Y} \\ &= \frac{dG_t}{Y} + (1+\rho)\frac{d\mathrm{B}_{t-1}}{Y} + (1+\rho)b\frac{d(1+i_{t-1})}{1+\rho} - b(1+\rho)d\Pi_t \\ &\quad -\frac{dT_t}{Y} - \frac{d(\Delta M_t/P_t)}{Y} \end{aligned} , \text{(3-36)}$$

where we use the definition $b \equiv \frac{\mathrm{B}}{Y}$ and the fact that $G = 0$.

Seignorage can be rewritten as:

$$\begin{aligned} \frac{\Delta M_t}{P_t} &= \frac{\Delta M_t}{M_{t-1}}\frac{P_{t-1}}{P_t}L_{t-1} \\ &= \frac{\Delta M_t}{M_{t-1}}\Pi_t^{-1}L_{t-1} \end{aligned} . \text{(3-37)}$$

Total derivative of Eq.(3-37) yields:

$$\begin{aligned} d\left(\Delta M_t/P_t\right) &= \frac{L}{M}d\Delta M_t - M^{-2}\Delta M L dM_{t-1} + \frac{\Delta M}{M}L(-1)d\Pi_t + \frac{\Delta M}{M}dL_{t-1} \\ &= \frac{L}{M}d\Delta M_t \end{aligned},$$

where we use the fact that $\Delta M = 0$. Dividing both sides of the previous expression yields by Y yields:

$$\frac{d\left(\Delta M_t/P_t\right)}{Y} = \frac{L}{Y}\frac{d\Delta M_t}{M}, \text{ (3-38)}$$

with $\chi \equiv \frac{L}{Y}$.

Plugging Eqs.(3-9) and (3-38) into Eq.(3-36) yields:

$$\hat{b}_t = \hat{g}_t + (1+\rho)\hat{b}_{t-1} + (1+\rho)b\hat{i}_{t-1} - b(1+\rho)\pi_t - \widehat{tr}_t - \chi\Delta m_t, \text{ (3-39)}$$

with $\hat{b}_t \equiv \frac{d\mathrm{B}_t}{Y}$, $\hat{g}_t \equiv \frac{dG_t}{Y}$ and $\widehat{tr}_t \equiv \frac{TR_t - TR}{Y}$. Eq.(3-39) is identical with Eq.(31) in the text.

A simple tax rule is given by:

$$\widehat{tr}_t = \psi_b\hat{b}_{t-1} + \hat{\varsigma}_t, \text{ (3-40)}$$

which is identical with Eq.(38) in the text.

Plugging Eq.(3-40) into Eq.(3-39) yields:

$$\hat{b}_t = (1+\rho-\psi_b)\hat{b}_{t-1} + (1+\rho)b\hat{i}_{t-1} - b(1+\rho)\pi_t + \hat{g}_t - \hat{\varsigma}_t - \chi\Delta m_t, \text{ (3-41)}$$

which is identical with Eq.(39) in the text.

3.10 Relationship between the CPI Inflation and GDP Inflation

Eq.(3-3) can be rewritten as:

$$\begin{aligned} p_t &= (1-\nu)p_{H,t} + \nu p_{F,t} \\ &= p_{H,t} + \nu s_t \end{aligned}.$$

First order differential equation of the previous expression is given by:

$$\pi_t = \pi_{H,t} + \nu\left(s_t - s_{t-1}\right),$$

which is identical with Eq.(32) in the text.

3.11 Trade balance

Total derivative of Eq.(1-34) is given by:

$$d\left(NX_t/P_{H,t}\right)=dY_t-\nu CdS_t-dC_t-dG_t\ .$$

By dividing both sides of the previous expression by Y yields:

$$\frac{d\left(NX_t/P_{H,t}\right)}{Y}=\frac{dY_t}{Y}-\nu dS_t-\frac{dC_t}{C}-\frac{dG_t}{Y},$$

which can be rewritten as:

$$\log\left[\frac{\left(NX_t/P_{H,t}\right)}{Y}\right]=\log\left(\frac{Y_t}{Y}\right)-\log g\left(S_t\right)-\log\left(\frac{C_t}{C}\right)-\log\left(\frac{G_t}{Y}\right).$$

Let define $\widehat{nx}_t\equiv\log\left[\frac{\left(NX_t/P_{H,t}\right)}{Y}\right]$, which is the ratio of trade balance to the GDP. Then the previous expression can be rewritten as:

$$\widehat{nx}_t=\hat{y}_t-\nu s_t-\hat{c}_t-\hat{g}_t\ , \text{(3-42)}$$

which is identical with Eq.(33) in the text.
Plugging Eqs.(3-7) and (3-10) into Eq.(3-42) yields:

$$\widehat{nx}_t=\frac{\nu(1-\nu)(\sigma-1)}{\sigma}s_t-\frac{\nu}{\sigma}\hat{\rho}_t\ ,$$

where we use $\zeta_t=\hat{\rho}_t$. Plugging $\sigma=1$ into the previous expression yields:

$$\widehat{nx}_t=-\nu\hat{\rho}_t\ ,$$

which implies that just the demand shock affects the trade balance under our benchmark parameterization. As long as the demand shock hit does not hit the economy, balanced trade attains (See Section 4.4 in the text).

4 Policy Regimes

Plugging $\hat{b}_t=0$ for all t into Eq.(3-41) yields:

$$\Delta m_t=\frac{1}{\chi}(1+\rho)b\hat{i}_{t-1}-\frac{b}{\chi}(1+\rho)\pi_t+\frac{1}{\chi}\hat{g}_t-\frac{1}{\chi}\hat{\varsigma}_t\ ,$$

which is identical with Eq.(40) in the text.

5 Some Entities

The domestic and the imported goods inflation is given by:

$$\begin{aligned} \pi_{H,t} &= p_{H,t} - p_{H,t-1} \\ \pi_{F,t} &= p_{F,t} - p_{F,t-1} \end{aligned}, \text{(5-1)}$$

which are Eqs(35) and(36) in the text, respectively.

The nominal exchange rate is calculated by:

$$s_t = e_t + p_t^* - p_{H,t},$$

which is identical with Eq.(34) in the text.

Subtracting the first equality in Eq.(5-1) from the second equality in Eq.(5-1) yields:

$$\begin{aligned} \pi_{F,t} - \pi_{H,t} &= p_{F,t} - p_{H,t} - \left(p_{F,t-1} - p_{H,t-1}\right) \\ &= s_t - s_{t-1} \end{aligned},$$

which can be rewritten as:

$\pi_{F,t} = s_t - s_{t-1} + \pi_{H,t}$. (5-2)

Eq.(5-2) is identical with Eq(37) in the text.

6 Introducing Imperfect Pass-through

6.1 International Risk Sharing Condition

Note that $Q_t \equiv \frac{E_t P_t^*}{P_t}$ can be rewritten as in the imperfect pass-through environment as follows:

$$\begin{aligned} Q_t &\equiv \frac{E_t P_t^*}{P_t} \\ &= \frac{E_t P_{F,t}^*}{P_t^{1-\nu} P_{F,t}^{\nu}} \\ &= \frac{P_{F,t}}{P_t^{1-\nu} P_{F,t}^{\nu}} \frac{E_t P_{F,t}^*}{P_{F,t}} \\ &= S_t^{1-\nu} \Psi_t \end{aligned},$$

which is identical with Eq.(A.1) in Appendix A. Plugging the previous expression into

Eq.(1-10) yields:

$$U_{c,t}^{-1}=\vartheta\left(U_{c,t}^{*}\right)^{-1}S_{t}^{1-\nu}\Psi_{t}\frac{Z_{t}}{Z_{t}^{*}}, \text{(6-1)}$$

which is identical with Eq.(A.2) in Appendix A.

6.2 Foreign Retailers

Consider a foreign exporter exporting good j at a cost (i.e., price paid in the world market) $E_t P_{F,t}^{*}(j)$. Like local producers, the same exporter faces a downward sloping demand for such goods and therefore chooses a price $\tilde{P}_{F,t}(j)$, expressed in units of domestic currency, to maximize:

$$\max_{\tilde{P}_{F,t}(j)}\sum_{k=0}^{\infty}\theta_F^k \mathrm{E}_t\left\{\Lambda_{t,t+k}^{*}\left(\frac{1}{P_{t+k}^{*}}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_t}-P_{F,t+k}^{*}(j)(1-\tau_F)\right]C_{F,t+k}(j)\right\},$$

with $C_{F,t+k}(j)\equiv\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+k}}\right)^{-\varepsilon}C_{F,t+k}$ where $\Lambda_{t,t+k}^{*}\equiv \mathrm{Q}_{t,t+k}^{*}\left(\frac{P_{t+k}^{*}}{P_t^{*}}\right)=\beta^k\left[\frac{\left(U_{c,t}^{*}\right)^{-1}}{\left(U_{c,t+k}^{*}\right)^{-1}}\frac{Z_{t+k}^{*}}{Z_t^{*}}\right]$

denotes the discount factor, $\mathrm{Q}_{t,t+k}^{*}$ denotes the price of a one period discount bond paying off one unit of foreign currency and τ_F denotes an export subsidiary. The previous expression can be rewritten as :

$$\max_{\tilde{P}_{F,t}(j)}\left\{\begin{array}{l}\Lambda_{t,t}^{*}\left(\frac{1}{P_t^{*}}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_t}\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon}C_{F,t}-P_{F,t}^{*}(j)(1-\tau_F)\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon}C_{F,t}\right]+\\ \theta_F\Lambda_{t,t+1}^{*}\left(\frac{1}{P_{t+1}^{*}}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_t}\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+1}}\right)^{-\varepsilon}C_{F,t+1}-P_{F,t+1}^{*}(j)(1-\tau_F)\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+1}}\right)^{-\varepsilon}C_{F,t+1}\right]\\ +\theta^2\Lambda_{t,t+2}^{*}\left(\frac{1}{P_{t+2}^{*}}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_t}\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+2}}\right)^{-\varepsilon}C_{F,t+2}-P_{F,t+1}^{*}(j)(1-\tau_F)\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+2}}\right)^{-\varepsilon}C_{F,t+2}\right]+\cdots\end{array}\right\}$$

The FONC for firms is given by:

$$
\begin{aligned}
&\Lambda^*_{t,t}\left(\frac{1}{P^*_t}\right)\left[(1-\varepsilon)\frac{\tilde{P}_{F,t}}{E_t}(j)^{-\varepsilon}P^{\varepsilon}_{F,t}C_{F,t}-P^*_{F,t}(j)(1-\tau_F)(-\varepsilon)\tilde{P}_{F,t}(j)^{-\varepsilon-1}P^{\varepsilon}_{F,t}C_{F,t}\right]\\
&+\theta\Lambda^*_{t,t+1}\left(\frac{1}{P^*_{t+1}}\right)\left[\begin{array}{l}(1-\varepsilon)\dfrac{\tilde{P}_{F,t}}{E_t}(j)^{-\varepsilon}P^{\varepsilon}_{F,t+1}C_{F,t+1}\\ -P^*_{F,t+1}(j)(1-\tau_F)(-\varepsilon)\tilde{P}_{F,t}(j)^{-\varepsilon-1}P^{\varepsilon}_{F,t+1}C_{F,t+1}\end{array}\right],\\
&+\theta^2\Lambda^*_{t,t+2}\left(\frac{1}{P^*_{t+2}}\right)\left[\begin{array}{l}(1-\varepsilon)\dfrac{\tilde{P}_{F,t}}{E_t}(j)^{-\varepsilon}P^{\varepsilon}_{F,t+2}C_{F,t+2}\\ -P^*_{F,t+2}(j)(1-\tau_F)(-\varepsilon)\tilde{P}_{F,t}(j)^{-\varepsilon}P^{\varepsilon}_{F,t+2}C_{F,t+2}\end{array}\right]+\cdots=0
\end{aligned}
$$

which can be rewritten as:

$$
\begin{aligned}
&\Lambda^*_{t,t}\left(\frac{1}{P^*_t}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_t}\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon}C_{F,t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t}(j)\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon}C_{F,t}\right]\\
&+\theta\Lambda^*_{t,t+1}\left(\frac{1}{P^*_{t+1}}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_t}\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+1}}\right)^{-\varepsilon}C_{F,t+1}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+1}(j)\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+1}}\right)^{-\varepsilon}C_{F,t+1}\right].\\
&+\theta^2\Lambda^*_{t,t+2}\left(\frac{1}{P^*_{t+2}}\right)\left[\frac{\tilde{P}_{F,t}(j)}{E_t}\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+2}}\right)^{-\varepsilon}C_{F,t+2}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+1}(j)\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+2}}\right)^{-\varepsilon}C_{F,t+2}\right]+\cdots=0
\end{aligned}
$$

By using the definition $C_{F,t+k|t}\equiv\left(\frac{\tilde{P}_{F,t}(j)}{P_{F,t+k}}\right)^{-\varepsilon}C_{F,t+k}$, the previous expression can be rewritten as:

$$
\begin{aligned}
&\Lambda^*_{t,t}\left(\frac{1}{P^*_t}\right)\left[\frac{\tilde{P}_{F,t}}{E_t}(j)C_{F,t|t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t}(j)C_{F,t|t}\right]\\
&+\theta\Lambda^*_{t,t+1}\left(\frac{1}{P^*_{t+1}}\right)\left[\frac{\tilde{P}_{F,t}}{E_t}(j)C_{F,t+1|t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+1}(j)C_{F,t+1|t}\right],\\
&+\theta^2\Lambda^*_{t,t+2}\left(\frac{1}{P^*_{t+2}}\right)\left[\frac{\tilde{P}_{F,t}}{E_t}(j)C_{F,t+2|t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+1}(j)C_{F,t+2|t}\right]+\cdots=0
\end{aligned}
$$

which can be rewritten as:

$$\Lambda^*_{t,t}\left(\frac{1}{P^*_t}\right)C_{F,t|t}\left[\frac{\tilde{P}_{F,t}(j)}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t}(j)\right]$$
$$+\theta\Lambda^*_{t,t+1}\left(\frac{1}{P^*_{t+1}}\right)C_{F,t+1|t}\left[\frac{\tilde{P}_{F,t}(j)}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+1}(j)\right] \quad .$$
$$+\theta^2\Lambda^*_{t,t+2}\left(\frac{1}{P^*_{t+2}}\right)C_{F,t+2|t}\left[\frac{\tilde{P}_{F,t}(j)}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+2}(j)\right]+\cdots=0$$

The previous expression can be compact expression as:

$$\sum_{k=0}^{\infty}\theta^k \mathrm{E}_t\left[\Lambda^*_{t,t+k}\left(\frac{1}{P^*_{t+k}}\right)C_{F,t+k|t}\left(\frac{\tilde{P}_{F,t}}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+k}\right)\right]=0 \text{ , (6-2)}$$

where we use the fact that $\tilde{P}_{F,t}(j)=\tilde{P}_{F,t}$ and $P^*_{F,t+k}(j)=P^*_{F,t+k}$ in the symmetric equilibrium. Eq.(6-2) is identical with Eq.(A.3) in Appendix A.

Plugging $\Lambda^*_{t,t+k}\equiv \mathrm{Q}^*_{t,t+k}\left(\frac{P^*_{t+k}}{P^*_t}\right)=\beta^k\left[\frac{\left(U^*_{c,t}\right)^{-1}}{\left(U^*_{c,t+k}\right)^{-1}}\frac{Z^*_{t+k}}{Z^*_t}\right]$ into Eq.(6-2) yields:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left[\left[\frac{\left(U^*_{c,t}\right)^{-1}}{\left(U^*_{c,t+k}\right)^{-1}}\frac{Z^*_{t+k}}{Z^*_t}\right]\left(\frac{1}{P^*_{t+k}}\right)C_{F,t+k|t}\left(\frac{\tilde{P}_{F,t}}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+k}\right)\right]=0.$$

By multiplying $U_{c,t}Z_t$ both sides of the previous expression yields:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{P^*_{t+k}\left(U^*_{c,t+k}\right)^{-1}}\right]Z^*_{t+k}C_{F,t+k|t}\left(\frac{\tilde{P}_{F,t}}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)P^*_{F,t+k}\right)\right\}=0.$$

Multiplying both sides of the previous expression by $\frac{1}{P_{F,t-1}/E_{t-1}}$ yields:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{P^*_{t+k}\left(U^*_{c,t+k}\right)^{-1}}\right]Z^*_{t+k}C_{F,t+k|t}\left[\frac{\tilde{P}_{F,t}/E_t}{P_{F,t-1}/E_{t-1}}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\frac{P^*_{F,t+k}}{P_{F,t+k}}\frac{P_{F,t+k}}{P_{F,t-1}/E_{t-1}}\right]\right\}=0,$$

which can be rewritten as:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{P^*_{t+k}\left(U^*_{c,t+k}\right)^{-1}}\right]Z^*_{t+k}C_{F,t+k|t}\left[\frac{\tilde{P}_{F,t}}{P_{F,t-1}}\frac{E_{t-1}}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\frac{E_{t+k}P^*_{F,t+k}}{P_{F,t+k}}\frac{E_{t-1}}{E_{t+k}}\frac{P_{F,t+k}}{P_{F,t-1}}\right]\right\}=0$$

Let define $\Psi_{t+k}\equiv\frac{E_{t+k}P^*_{F,t+k}}{P_{F,t+k}}$ and $\Pi_{F/E,t-1,t+k}\equiv\frac{P_{F,t+k}}{P_{F,t-1}}\frac{E_{t-1}}{E_{t+k}}$. Then, the previous expression can be rewritten as:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{P_{t+k}^*\left(U_{c,t+k}^*\right)^{-1}}\right]Z_{t+k}^* C_{F,t+k|t}\left[\frac{\tilde{P}_{F,t}}{P_{F,t-1}}\frac{E_{t-1}}{E_t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\Psi_{t+k}\Pi_{F/E,t-1,t+k}\right]\right\}=0$$

Let define $\tilde{X}_{F/E,t} \equiv \frac{\tilde{P}_{F,t}}{P_{F,t-1}}\frac{E_{t-1}}{E_t}$. Then the previous expression can be rewritten as:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{P_{t+k}^*\left(U_{c,t+k}^*\right)^{-1}}\right]Z_{t+k}^* C_{F,t+k|t}\left[\tilde{X}_{F/E,t}-\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\Psi_{t+k}\Pi_{F/E,t-1,t+k}\right]\right\}=0,$$

which can be rewritten as:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{\left(U_{c,t+k}^*\right)^{-1}}\right]Z_{t+k}^* C_{F,t+k|t}\frac{P_{F,t+k}}{E_{t+k}P_{t+k}^*}\frac{E_{t+k}}{P_{F,t+k}}\frac{P_{t-1}}{E_{t-1}}\left[\begin{array}{l}\tilde{X}_{F/E,t}\\ -\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\Psi_{t+k}\Pi_{F/E,t-1,t+k}\end{array}\right]\right\}=0.$$

By using the definition $\Psi_{t+k} \equiv \frac{E_{t+k}P_{F,t+k}^*}{P_{F,t+k}}$ and $\Pi_{F/E,t-1,t+k} \equiv \frac{P_{F,t+k}}{P_{F,t-1}}\frac{E_{t-1}}{E_{t+k}}$, the previous expression can be rewritten as:

$$\sum_{k=0}^{\infty}(\theta\beta)^k \mathrm{E}_t\left\{\left[\frac{1}{\left(U_{c,t+k}^*\right)^{-1}}\right]Z_{t+k}^* C_{F,t+k|t}\Psi_{t+k}^{-1}\Pi_{F/E,t-1,t+k}^{-1}\left[\begin{array}{l}\tilde{X}_{F/E,t}\\ -\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\Psi_{t+k}\Pi_{F/E,t-1,t+k}\end{array}\right]\right\}=0.$$

The previous compact form can be rewritten as:

$$\begin{aligned}
&\left[\left(U_{c,t}^*\right)^{-1}\right]^{-1} Z_t^* C_{F,t|t}\Psi_t^{-1}\Pi_{F/E,t}^{-1}\tilde{X}_{F/E,t}\\
&+\theta\beta\left[\left(U_{c,t+1}^*\right)^{-1}\right]^{-1} Z_{t+1}^* C_{F,t+1|t}\Psi_{t+1}^{-1}\Pi_{F/E,t+1}^{-1}\Pi_{F/E,t}^{-1}\tilde{X}_{F,t}\\
&+(\theta\beta)^2\left[\left(U_{c,t+2}^*\right)^{-1}\right]^{-1} Z_{t+2}^* C_{F,t+2|t}\Psi_{t+2}^{-1}\Pi_{F/E,t+2}^{-1}\Pi_{F/E,t+1}^{-1}\Pi_{F/E,t}^{-1}\tilde{X}_{F,t}\\
&+\cdots=\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\left[\left(U_{c,t}^*\right)^{-1}\right]^{-1} Z_t^* C_{F,t|t}\\
&+\theta\beta\left[\left(U_{c,t+1}^*\right)^{-1}\right]^{-1} Z_{t+1}^* C_{F,t+1|t}\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\\
&+(\theta\beta)^2\left[\left(U_{c,t+2}^*\right)^{-1}\right]^{-1} Z_{t+2}^* C_{F,t+2|t}\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)+\cdots
\end{aligned},$$

with $\Pi_{F/E,t} \equiv \frac{P_{F,t}/E_t}{P_{F,t-1}/E_{t-1}} = \frac{P_{F,t}}{E_t}\frac{E_{t-1}}{P_{F,t-1}}$. Rearranging the previous expression yields:

$$\tilde{X}_{F/E,t}\left\{\begin{array}{l}\left[\left(U^*_{c,t}\right)^{-1}\right]^{-1} Z^*_t C_{F,t|t}\Psi_t^{-1}\Pi^{-1}_{F/E,t} \\ +\theta\beta\left[\left(U^*_{c,t+1}\right)^{-1}\right]^{-1} Z^*_{t+1} C_{F,t+1|t}\Psi_{t+1}^{-1}\Pi^{-1}_{F/E,t+1}\Pi^{-1}_{F/E,t} \\ +(\theta\beta)^2\left[\left(U^*_{c,t+2}\right)^{-1}\right]^{-1} Z^*_{t+2} C_{F,t+2|t}\Psi_{t+2}^{-1}\Pi^{-1}_{F/E,t+2}\Pi^{-1}_{F/E,t+1}\Pi^{-1}_{F/E,t} \\ +\cdots\end{array}\right\},$$

$$=\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\left\{\begin{array}{l}\left[\left(U^*_{c,t}\right)^{-1}\right]^{-1} Z^*_t C_{F,t|t} \\ +\theta\beta\left[\left(U^*_{c,t+1}\right)^{-1}\right]^{-1} Z^*_{t+1} C_{F,t+1|t}\frac{\varepsilon}{\varepsilon-1}(1-\tau_F) \\ +(\theta\beta)^2\left[\left(U^*_{c,t+2}\right)^{-1}\right]^{-1} Z^*_{t+2} C_{F,t+2|t}\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)+\cdots\end{array}\right\}$$

which can be rewritten as:

$$\tilde{X}_{F/E,t}=\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\left\{\begin{array}{l}\left[\left(U^*_{c,t}\right)^{-1}\right]^{-1} Z^*_t C_{F,t|t} \\ +\theta_F\beta\left[\left(U^*_{c,t+1}\right)^{-1}\right]^{-1} Z^*_{t+1} C_{F,t+1|t}\frac{\varepsilon}{\varepsilon-1}(1-\tau_F) \\ +(\theta_F\beta)^2\left[\left(U^*_{c,t+2}\right)^{-1}\right]^{-1} Z^*_{t+2} C_{F,t+2|t}\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)+\cdots\end{array}\right\}$$

$$\times\left\{\begin{array}{l}\left[\left(U^*_{c,t}\right)^{-1}\right]^{-1} Z^*_t C_{F,t|t}\Psi_t^{-1}\Pi^{-1}_{F/E,t} \\ +\theta_F\beta\left[\left(U^*_{c,t+1}\right)^{-1}\right]^{-1} Z^*_{t+1} C_{F,t+1|t}\Psi_{t+1}^{-1}\Pi^{-1}_{F/E,t+1}\Pi^{-1}_{F/E,t} \\ +(\theta_F\beta)^2\left[\left(U^*_{c,t+2}\right)^{-1}\right]^{-1} Z^*_{t+2} C_{F,t+2|t}\Psi_{t+2}^{-1}\Pi^{-1}_{F/E,t+2}\Pi^{-1}_{F/E,t+1}\Pi^{-1}_{F/E,t} \\ +\cdots\end{array}\right\}^{-1} \quad (6\text{-}3)$$

or:

$$\tilde{X}_{F/E,t}=\frac{\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\sum_{k=0}^{\infty}(\theta\beta)^k\left[\left(U^*_{c,t+k}\right)^{-1}\right]^{-1} Z^*_{t+k} C_{F,t+k|t}}{\sum_{k=0}^{\infty}(\theta\beta)^k\left[\left(U^*_{c,t+k}\right)^{-1}\right]^{-1} Z^*_{t+k} C_{F,t+k|t}\Psi_{t+k}^{-1}\prod_{h=0}^{k}\Pi^{-1}_{F/E,t+h}}. \quad (6\text{-}4)$$

6.3 Market Clearing Condition

Demands for export in the PTM environment is given by:

$$
\begin{aligned}
EX_t &= \nu\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-1} C_t^* \\
&= \nu\left(\frac{P_{H,t}^*}{P_{F,t}^*}\right)^{-1} C_t^* \\
&= \nu\left(\frac{P_{H,t}}{E_t P_{F,t}^*}\right)^{-1} C_t^* \qquad , (6\text{-}5) \\
&= \nu\left(\frac{P_{H,t}}{P_{F,t}}\frac{P_{F,t}}{E_t P_{F,t}^*}\right)^{-1} C_t^* \\
&= \nu\left(\frac{P_{F,t}}{P_{H,t}}\right)\left(\frac{E_t P_{F,t}^*}{P_{F,t}}\right) C_t^* \\
&= \nu S_t \Psi_t Y_t^*
\end{aligned}
$$

which is identical with Eq.(A.4) in Appendix A where we use the definition of LOOP gap

$\Psi_t \equiv \frac{E_t P_{F,t}^*}{P_{F,t}}$ as well as $C_t^* = Y_t^*$.

Plugging Eqs.(1-12), (1-17), (1-20), (1-27), (1-26) and (6-5) into Eq.(1-25) yields:

$$
\begin{aligned}
\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} Y_t &= \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} + \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\varepsilon} EX_t + \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon} G_t \\
&= (1-\nu)\left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon} S_t^{\nu} C_t + \nu\left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\varepsilon} S_t \Psi_t Y_t^* + \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon} G_t \\
&= \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon}\left[(1-\nu) S_t^{\nu} C_t + \nu S_t \Psi_t Y_t^* + G_t\right] \qquad , \\
&= \left(\frac{P_t(j)}{P_{H,t}}\right)^{-\varepsilon}\left[(1-\nu) S_t^{\nu} C_t + \nu S_t \Psi_t Y_t^* + G_t\right]
\end{aligned}
$$

where we use the LOOP implying that $P_{H,t}^*(j) = \frac{P_{H,t}(j)}{E_t}$ and $P_{H,t}^* = \frac{P_{H,t}}{E_t}$. By dividing both sides of the previous expression by $\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}$ yields:

$Y_t = (1-\nu) S_t^{\nu} C_t + \nu S_t \Psi_t C_t^* + G_t$, (6-6)

which is identical with Eq.(A.5) in Appendix A.

6.4 The Steady State

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_{H,t} = \Pi_t = 1$. Further, we assume $Z_t = Z_t^* = 1$ and $G_t = 0$.

Eqs.(1-6) and (1-9) implies as follows:

$$\beta = \frac{1}{1+i} = \frac{1}{1+i^*},$$

which is identical with Eq.(2-1).

Eq.(1-7) implies that:

$$\frac{W}{P} = \frac{V_n}{U_c},$$

which is identical with Eq.(2-2).

Eq.(1-8) implies as follows:

$$\frac{U_l}{U_c} = \beta i,$$

which is identical with Eq.(2-3).

Eq.(6-4) implies:

$$1 = \frac{\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]\left[\left(U_c^*\right)^{-1}\right]^{-1} C_F}{\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]\left[\left(U_c^*\right)^{-1}\right]^{-1} C_F \Psi^{-1}},$$

which can be rewritten as:

$$\Psi = \left[\mathrm{M}(1-\tau_F)\right]^{-1},$$

with $\mathrm{M} \equiv \frac{\varepsilon}{\varepsilon-1}$ being the constant markup. As long as we assume $\mathrm{M}(1-\tau_F) = 1$,

$\Psi = 1$, (6-7)

which implies that $EP_F^* = P_F$ is applicable. Eq.(6-7) is identical with Eq.(B.1) in Appendix B.

Due to Eq.(6-7), $Q = S^{1-\nu}$ is applicable, then the other steady state conditions are identical to those in the perfect pass-through environment, namely, Section 2 in this appendix.

6.5 Log-linearization of the Model

6.5.1 Log-linearizing the International Risk Sharing Condition

Total derivative of the definition of the TOT is given by:

$$\begin{aligned} dS_t &= \frac{1}{P_H} dP_{F,t} + P_F(-)P_H^{-2} dP_{H,t} \\ &= \frac{P_F}{P_H}\frac{dP_{F,t}}{P_F} - \frac{P_F}{P_H}\frac{dP_{H,t}}{P_H} \\ &= S\frac{dP_{F,t}}{P_F} - S\frac{dP_{H,t}}{P_H} \end{aligned} .$$

Dividing both sides of the previous expression yields:

$$\frac{dS_t}{S} = \frac{dP_{F,t}}{P_F} - \frac{dP_{H,t}}{P_H},$$

Which can be expressed as:

$s_t = p_{F,t} - p_{H,t}$.

Total derivative of the definition of the real exchange rate

$Q_t \equiv \frac{E_t P_t^*}{P_t} = \frac{E_t P_{F,t}^*}{P_{F,t}} \frac{P_{F,t}}{P_{H,t}^{1-\nu} P_{F,t}^{\nu}} = \Psi_t S_t^{1-\nu}$ is given by:

$$\begin{aligned} dQ_t &= S^{1-\nu} d\Psi_t + \Psi(1-\nu)S^{-\nu} dS_t \\ &= S^{1-\nu}\frac{d\Psi_t}{\Psi} + (1-\nu)S^{-\nu}\frac{dS_t}{S} \end{aligned},$$

which can be rewritten as:

$q_t = \psi_t + (1-\nu)s_t$. (6-8)

Plugging Eq.(6-8) into Eq.(3-1) yields:

$$\hat{\xi}_t = -\psi_t - (1-\nu)s_t + \hat{\xi}_t^* - \zeta_t$$

which is log-linearized international risk sharing condition and is identical with Eq.(51) in the text.

6.5.2 Log-linearizing the Market Clearing Condition

Total derivative of Eq.(6-6) is given by:

$$\begin{aligned} dY_t &= \left[(1-\nu)\nu C + \nu Y^*\right] dS_t + (1-\nu)dC_t + \nu Y^* d\Psi_t + \nu dY_t^* + dG_t \\ &= \nu\left[(1-\nu)+1\right] Y dS_t + (1-\nu)dC_t + \nu Y^* d\Psi_t + \nu dY_t^* + dG_t \\ &= \nu(2-\nu) Y dS_t + (1-\nu)dC_t + \nu Y^* d\Psi_t + \nu dY_t^* + dG_t \end{aligned} 。$$

By dividing both sides of Eq.(2-3-8) by Y, we have:

$$\frac{dY_t}{Y}=\nu(2-\nu)dS_t+(1-\nu)\frac{dC_t}{C}+\nu d\Psi_t+\nu\frac{dY_t^*}{Y^*}+\frac{dG_t}{Y},$$

which can be rewritten as:

$$\log\left(\frac{Y_t}{Y}\right)=\nu(2-\nu)\log S_t+(1-\nu)\log\left(\frac{C_t}{C}\right)+\nu\log\Psi_t+\nu\log\left(\frac{Y_t^*}{Y^*}\right)+\log\left(\frac{G_t}{Y}\right),$$

The previous expression can be rewritten as:

$$\hat{y}_t=\nu(2-\nu)s_t+(1-\nu)\hat{c}_t+\nu\psi_t+\nu\hat{y}_t^*+\hat{g}_t,$$

which is identical with Eq.(52) in the text.

6.5.3 Deriving the Import Goods Inflation Equation in Imperfect Pass-through Environment

$$\tilde{X}_{F,t}\left\{\begin{array}{l}\left[\left(U_{c,t}^*\right)^{-1}\right]^{-1}Z_t^*C_{F,t|t}\Psi_t^{-1}\Pi_{F,t}^{-1}\\+\theta_F\beta\left[\left(U_{c,t+1}^*\right)^{-1}\right]^{-1}Z_{t+1}^*C_{F,t+1|t}\dfrac{E_{t+1}}{E_t}\Psi_{t+1}^{-1}\Pi_{F,t+1}^{-1}\Pi_{F,t}^{-1}\\+(\theta_F\beta)^2\left[\left(U_{c,t}^*\right)^{-1}\right]^{-1}Z_{t+2}^*C_{F,t+2|t}\dfrac{E_{t+2}}{E_{t+1}}\dfrac{E_{t+1}}{E_t}\Psi_{t+2}^{-1}\Pi_{F,t+2}^{-1}\Pi_{F,t+1}^{-1}\Pi_{F,t}^{-1}\\+\cdots\end{array}\right\}$$

$$=\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\left\{\begin{array}{l}\left[\left(U_{c,t}^*\right)^{-1}\right]^{-1}Z_t^*C_{F,t|t}+\theta_F\beta\left[\left(U_{c,t}^*\right)^{-1}\right]^{-1}Z_{t+1}^*C_{F,t+1|t}\\+(\theta_F\beta)^2\left[\left(U_{c,t}^*\right)^{-1}\right]^{-1}Z_{t+2}^*C_{F,t+2|t}+\cdots\end{array}\right\}$$

Total derivative of Eq.(6-3) is given by:

$$
\begin{aligned}
d\tilde{X}_{F/E,t} &= \left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]U_c^*C_F(-1)\left\{\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]U_c^*C_F\right\}^{-2} \\
&\times\left\{\begin{array}{l}
C_F dU_{c,t}^* + \theta_F\beta C_F dU_{c,t+1}^* + (\theta_F\beta)^2 C_F dU_{c,t+2}^* + \cdots \\
+U_c^*C_F dZ_t^* + \theta_F\beta U_c^*C_F dZ_{t+1}^* + (\theta_F\beta)^2 U_c^*C_F dZ_{t+2}^* + \cdots \\
+U_c^* dC_{F,t|t} + \theta_F\beta U_c^* dC_{F,t+1|t} + (\theta_F\beta)^2 U_c^* dC_{F,t+2|t} + \cdots \\
+(-1)U_c^*C_F d\Psi_t + \theta_F\beta(-1)U_c^*C_F d\Psi_{t+1} + (\theta_F\beta)^2(-1)U_c^*C_F d\Psi_{t+2} + \cdots \\
+(-1)U_c^*C_F\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t} \\
+(-1)U_c^*C_F\left[\theta_F\beta+(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t+1} + (-1)U_c^*C_F\left[(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t+2} \\
+\cdots
\end{array}\right\} \\
&+\left\{\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]U_c^*C_F\right\}^{-1} \\
&\times\left[\begin{array}{l}
C_F dU_{c,t}^* + \theta_F\beta C_F dU_{c,t+1}^* + (\theta_F\beta)^2 C_F dU_{c,t+2}^* + \cdots \\
+U_c^*C_F dZ_t^* + \theta_F\beta U_c^*C_F dZ_{t+1}^* + (\theta_F\beta)^2 U_c^*C_F dZ_{t+2}^* + \cdots \\
+U_c^* dC_{F,t|t} + \theta_F\beta U_c^* dC_{F,t+1|t} + (\theta_F\beta)^2 U_c^* dC_{F,t+2|t} + \cdots
\end{array}\right],
\end{aligned}
$$

which can be rewritten as:

$$
\begin{aligned}
d\tilde{X}_{F/E,t} &= \left\{\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]U_c^*C_F\right\}^{-1} \\
&\times\left\{\begin{array}{l}
U_c^*C_F d\Psi_t + \theta_F\beta(-1)U_c^*C_F d\Psi_{t+1} + (\theta_F\beta)^2(-1)U_c^*C_F d\Psi_{t+2} + \cdots \\
+U_c^*C_F\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t} \\
+U_c^*C_F\left[\theta_F\beta+(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t+1} + U_c^*C_F\left[(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t+2} \\
+\cdots
\end{array}\right\}.
\end{aligned}
$$

Further:

$$
d\tilde{X}_{F/E,t} = \left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]^{-1}\left\{\begin{array}{l}
d\Psi_t + \theta_F\beta d\Psi_{t+1} + (\theta_F\beta)^2 d\Psi_{t+2} + \cdots \\
+\left[1+\theta_F\beta+(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t} \\
+\left[\theta_F\beta+(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t+1} + \left[(\theta_F\beta)^2+\cdots\right]d\Pi_{F/E,t+2} \\
+\cdots
\end{array}\right\}.
$$

Note that $1+\theta\beta+(\theta\beta)^2+\cdots = \dfrac{1}{1-\theta\beta}$. Then, the previous expression can be rewritten as:

$$
\begin{aligned}
d\tilde{X}_{F/E,t} &= (1-\theta_F\beta)\left\{\begin{aligned} &d\Psi_t + \theta_F\beta d\Psi_{t+1} + (\theta_F\beta)^2 d\Psi_{t+2} + \cdots \\ &+\frac{1}{1-\theta_F\beta} d\Pi_{F/E,t} + \frac{\theta_F\beta}{1-\theta_F\beta} d\Pi_{F/E,t+1} + \frac{(\theta_F\beta)^2}{1-\theta_F\beta} d\Pi_{F/E,t+2} + \cdots \end{aligned}\right\} \\
&= (1-\theta_F\beta)\left[d\Psi_t + \theta_F\beta d\Psi_{t+1} + (\theta_F\beta)^2 d\Psi_{t+2} + \cdots\right] \\
&\quad + d\Pi_{F/E,t} + \theta_F\beta d\Pi_{F/E,t+1} + (\theta_F\beta)^2 d\Pi_{F/E,t+2} + \cdots
\end{aligned}
. \text{(6-9)}
$$

The definition of $\tilde{X}_{F/E,t}$ can be log-linearized as:

$$
\begin{aligned}
d\tilde{X}_{F/E,t} &= d\left(\frac{\tilde{P}_{F,t}}{P_{F,t-1}}\right) - d\left(\frac{E_t}{E_{t-1}}\right) \\
&= d\left(\frac{\tilde{P}_{F,t}}{P_F}\frac{P_F}{P_{F,t-1}}\right) - d\left(\frac{E_t}{E}\frac{E}{E_{t-1}}\right) \\
&= d\left(\frac{\tilde{P}_{F,t}}{P_F}\right) - d\left(\frac{P_{F,t-1}}{P_F}\right) - \left[d\left(\frac{E_t}{E}\right) - d\left(\frac{E_{t-1}}{E}\right)\right]
\end{aligned}
.
$$

Plugging the previous expression into Eq.(6-9) yields:

$$
\begin{aligned}
\tilde{p}_{F,t} - p_{F,t-1} - (e_t - e_{t-1}) &= (1-\theta_F\beta)\left[\psi_t + \theta_F\beta\psi_{t+1} + (\theta_F\beta)^2\psi_{t+2} + \cdots\right] \\
&\quad + \left[\pi_{F/E,t} + \theta_F\beta\pi_{F/E,t+1} + (\theta_F\beta)^2\pi_{F/E,t+2} + \cdots\right]
\end{aligned}
, \text{(6-10)}
$$

with $\pi_{F/E,t} \equiv \log\Pi_{F/E,t}$, $\tilde{p}_{F,t} \equiv \log\left(\frac{\tilde{P}_{F,t}}{P_F}\right)$, $p_{F,t} \equiv \log\left(\frac{P_{F,t}}{P_F}\right)$, $\Delta e_t \equiv e_t - e_{t-1}$ and $e_t \equiv \log E_t$.

Forwarding Eq.(6-10) one period yields:

$$
\begin{aligned}
\tilde{p}_{F,t+1} - p_{F,t} - (e_{t+1} - e_t) &= (1-\theta_F\beta)\left[\psi_{t+1} + \theta_F\beta\psi_{t+2} + (\theta_F\beta)^2\psi_{t+3} + \cdots\right] \\
&\quad + \left[\pi_{F/E,t+1} + \theta_F\beta\pi_{F/E,t+2} + (\theta_F\beta)^2\pi_{F/E,t+3} + \cdots\right]
\end{aligned}
.
$$

Multiplying $\theta_F\beta$ on both sides of the previous expression yields:

$$
\begin{aligned}
\theta_F\beta\left[\tilde{p}_{F,t+1} - p_{F,t} - (e_{t+1} - e_t)\right] &= (1-\theta_F\beta)\left[\theta_F\beta\psi_{t+1} + (\theta_F\beta)^2\psi_{t+2} + (\theta_F\beta)^3\psi_{t+3} + \cdots\right] \\
&\quad + \left[\theta_F\beta\pi_{F/E,t+1} + (\theta_F\beta)^2\pi_{F/E,t+2} + (\theta_F\beta)^3\pi_{F/E,t+3} + \cdots\right]
\end{aligned}
. \text{(6-11)}
$$

Eq.(6-10) can be rewritten as:

$$\begin{aligned}\tilde{p}_{F,t} - p_{F,t-1} - (e_t - e_{t-1}) = & (1-\theta_F\beta)\psi_t + \pi_{F/E,t} \\ & + (1-\theta_F\beta)\left[\theta_F\beta\psi_{t+1} + (\theta_F\beta)^2\psi_{t+2} + \cdots\right] \\ & + \left[\theta_F\beta\pi_{F/E,t+1} + (\theta_F\beta)^2\pi_{F/E,t+2} + \cdots\right]\end{aligned}.$$

Plugging Eq.(6-11) into the previous expression yields:

$$\tilde{p}_{F,t} - p_{F,t-1} - (e_t - e_{t-1}) = (1-\theta_F\beta)\psi_t + \pi_{F/E,t} + \theta_F\beta\left[\tilde{p}_{F,t+1} - p_{F,t} - (e_{t+1} - e_t)\right]. \text{ (6-12)}$$

Calvo-pricing's transitory equation is given by:

$$P_{F,t} = \left[\theta_F P_{F,t-1}^{1-\varepsilon} + (1-\theta_F)\tilde{P}_{F,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Log-linearizing the previous expression around the steady state yields:

$$p_{F,t} = \theta_F p_{F,t-1} + (1-\theta_F)\tilde{p}_{F,t}.$$

Subtracting $p_{F,t-1}$ from the both sides of the previous expression yields:

$$\pi_{F,t} = (1-\theta_F)(\tilde{p}_{F,t} - p_{F,t-1}),$$

which can be rewritten as:

$$\tilde{p}_{F,t} - p_{F,t-1} = \frac{1}{1-\theta_F}\pi_{F,t}. \text{ (6-13)}$$

Plugging Eq.(6-13) into Eq.(6-12) yields:

$$\frac{1}{1-\theta_F}\pi_{F,t} - (e_t - e_{t-1}) = (1-\theta_F\beta)\psi_t + \pi_{F/E,t} + \theta_F\beta\left[\frac{1}{1-\theta_F}\pi_{F,t+1} - (e_{t+1} - e_t)\right]. \text{ (6-14)}$$

Log-linearizing the definition of $\Pi_{F/E,t} \equiv \frac{P_{F,t}/E_t}{P_{F,t-1}/E_{t-1}} = \frac{P_{F,t}}{E_t}\frac{E_{t-1}}{P_{F,t-1}} = \Pi_{F,t}\left(\frac{E_t}{E_{t-1}}\right)^{-1}$ yields:

$$\begin{aligned}d\Pi_{F/E,t} &= d\Pi_{F,t} - \frac{dE_t}{E} + (-1)E(-1)E^{-2}dE_{t-1} \\ &= d\Pi_{F,t} - \frac{dE_t}{E} + \frac{dE_{t-1}}{E}\end{aligned}.$$

Thus, Eq.(6-14) can be rewritten as:

$$\frac{1}{1-\theta_F}\pi_{F,t} - (e_t - e_{t-1}) = (1-\theta_F\beta)\psi_t + \pi_{F,t} - (e_t - e_{t-1}) + \theta_F\beta\left[\frac{1}{1-\theta_F}\pi_{F,t+1} - (e_{t+1} - e_t)\right],$$

which can be rewritten as:

$$\left(\frac{1}{1-\theta_F}-1\right)\pi_{F,t}=(1-\theta_F\beta)\psi_t+\theta_F\beta\left[\frac{1}{1-\theta_F}\pi_{F,t+1}-(e_{t+1}-e_t)\right].$$

Note that $\frac{1}{1-\theta_F}-1=\frac{1-(1-\theta_F)}{1-\theta_F}=\frac{\theta_F}{1-\theta_F}$. Thus, the previous expression can be rewritten as:

$$\pi_{F,t}=\frac{1-\theta_F}{\theta_F}\left\{(1-\theta_F\beta)\psi_t+\theta_F\beta\left[\frac{1}{1-\theta_F}\pi_{F,t+1}-(e_{t+1}-e_t)\right]\right\}.$$

Finally, the previous expression can be rewritten as:

$$\pi_{F,t}=\beta\pi_{F,t+1}+\frac{(1-\theta_F)(1-\theta_F\beta)}{\theta_F}\psi_t-\frac{\beta(1-\theta_F)}{\theta_F}(e_{t+1}-e_t),$$

which is Eq.(53) in the text.

Plugging $\pi_{F/E,t}=\pi_{F,t}-(e_t-e_{t-1})$ into Eq.(6-10) yields:

$$\begin{aligned}\tilde{p}_{F,t}-p_{F,t-1}-(e_t-e_{t-1}) &=(1-\theta_F\beta)\left[\psi_t+\theta_F\beta\psi_{t+1}+(\theta_F\beta)^2\psi_{t+2}+\cdots\right]+\pi_{F,t}-(e_t-e_{t-1})\\ &\quad+\theta_F\beta\left[\pi_{F,t+1}-(e_{t+1}-e_t)\right]+(\theta_F\beta)^2\left[\pi_{F,t+2}-(e_{t+2}-e_{t+1})\right]+\cdots\\ &=(1-\theta_F\beta)\left[\psi_t+\theta_F\beta\psi_{t+1}+(\theta_F\beta)^2\psi_{t+2}+\cdots\right]\\ &\quad+(p_{F,t}-p_{F,t-1})-(e_t-e_{t-1})+\theta_F\beta\left[(p_{F,t+1}-p_{F,t})-(e_{t+1}-e_t)\right]\\ &\quad+(\theta_F\beta)^2\left[(p_{F,t+2}-p_{F,t+1})-(e_{t+2}-e_{t+1})\right]+\cdots\\ &=(1-\theta_F\beta)\left[\psi_t+\theta_F\beta\psi_{t+1}+(\theta_F\beta)^2\psi_{t+2}+\cdots\right]\\ &\quad-p_{F,t-1}+(1-\theta_F\beta)p_{F,t}+(1-\theta_F\beta)\theta_F\beta p_{F,t+1}+(1-\theta_F\beta)(\theta_F\beta)^2p_{F,t+2}\\ &\quad+\cdots\\ &\quad+e_{t-1}-(1-\theta_F\beta)e_t-(1-\theta_F\beta)\theta_F\beta e_{t+1}-(1-\theta_F\beta)(\theta_F\beta)^2e_{t+2}-\cdots\\ &=(1-\theta_F\beta)\begin{bmatrix}\psi_t+\theta_F\beta\psi_{t+1}+(\theta_F\beta)^2\psi_{t+2}+\cdots\\ +p_{F,t}+\theta_F\beta p_{F,t+1}+(\theta_F\beta)^2p_{F,t+2}+\cdots\\ -e_t-\theta_F\beta e_{t+1}-(\theta_F\beta)^2e_{t+2}-\cdots\end{bmatrix}-p_{F,t-1}+e_{t-1}\end{aligned},$$

Rearranging the previous expression on $\tilde{p}_{F,t}-e_t$ yields:

$$\tilde{p}_{F,t}-e_t=(1-\theta_F\beta)\sum_{k=0}^{\infty}(\theta_F\beta)^k\left[\psi_{t+k}+(p_{F,t+k}-e_{t+k})\right],$$

which is identical with log-linearized FONC for foreign retailers in Section 6.5.1 in the text.

6.5.4 Some Entities

The nominal exchange rate is calculated by:

$$s_t = e_t + p_t^* - \psi_t - p_{H,t} .$$

which is Eq.(54) in the text.

The LOOP gap is calculated via:

$$e_t = \psi_t + p_{F,t} - p_t^* ,$$

which is Eq.(55) in the text.

6.5.5 The Steady State in the Case of No Subsidiary

We focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which $\Pi_{H,t} = \Pi_t = 1$. Further, we assume $Z_t = Z_t^* = 1$ and $G_t = 0$.

Eqs.(1-6) and (1-9) implies as follows:

$$\begin{aligned}\beta &= \frac{1}{1+i} \\ &= \frac{1}{1+i^*}\end{aligned},$$

which is identical with Eq.(2-1).

Eq.(1-7) implies that:

$$\frac{W}{P} = \frac{V_n}{U_c} ,$$

which is identical with Eq.(2-2).

Eq.(1-8) implies as follows:

$$\frac{U_l}{U_c} = \beta i ,$$

which is identical with Eq.(2-3).

Eq.(6-4) implies:

$$1 = \frac{\frac{\varepsilon}{\varepsilon-1}(1-\tau_F)\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]\left[\left(U_c^*\right)^{-1}\right]^{-1} C_F}{\left[1+\theta\beta+(\theta\beta)^2+\cdots\right]\left[\left(U_c^*\right)^{-1}\right]^{-1} C_F \Psi^{-1}} ,$$

which can be rewritten as:

$$\Psi = \left[\mathrm{M}(1-\tau_F)\right]^{-1} ,$$

with $\mathrm{M} \equiv \frac{\varepsilon}{\varepsilon - 1}$ being the constant markup. As long as we assume $\tau_F = 0$,

$\Psi = \mathrm{M}^{-1}$. (6-15)

Eq.(1-24) implies:

$$MC = \frac{1}{1-\alpha}\frac{W}{P_H}N^{\alpha},$$

Which is identical with Eq.(2-5).

Eq.(1-16) can be rewritten as:

$$\frac{V_n}{U_c} = \frac{W}{P_H}\frac{P_H}{P},$$

which is identical with Eq.(2-6).

Plugging Eq.(2-5) into Eq.(2-6) yields:

$$\frac{V_n}{U_c} = \frac{1-\alpha}{N^{\alpha}\mathrm{M}}\frac{P_H}{P},$$

which is identical with Eq.(2-7).

Plugging Eq.(2-8) into Eq.(2-7) yields:

$$\frac{V_n}{U_c} = \frac{1-\alpha}{N^{\alpha}\mathrm{M}S^{\nu}},$$

which can be written as:

$$V_n = \frac{1-\alpha}{N^{\alpha}\mathrm{M}S^{\nu}}U_c,$$

which is identical with Eq.(2-9)

Eq.(6-1) implies:

$$U_c^{-1} = \vartheta\left(U_c^{*}\right)^{-1}S^{1-\nu}\Psi.$$

Plugging Eq.(6-15) into the previous expression yields:

$$\begin{aligned} U_c^{-1} &= \vartheta\left(U_c^{*}\right)^{-1}S^{1-\nu}\mathrm{M}^{-1} \\ &= \vartheta\left(U_c^{*}\right)^{-1}\omega(S) \end{aligned} \quad . \text{(6-16)}$$

Note that:

$$\begin{aligned} \omega(S) &\equiv Q \\ &= \frac{EP^{*}}{P} = \frac{P_F}{P_H^{1-\nu}P_F^{\nu}}\frac{EP_F^{*}}{P_F} = \left(\frac{P_F}{P_H}\right)^{1-\nu}\mathrm{M}^{-1} \\ &= S^{1-\nu}\mathrm{M}^{-1} \end{aligned} \quad . \text{(6-17)}$$

Eq.(6-16) can be rewritten as:

$$S^{\nu} = \vartheta\left(U_c^*\right)^{-1} S\mathrm{M}^{-1}U_c$$

Plugging the previous expression into Eq.(2-9) yields:

$$\begin{aligned} V_n &= \frac{1-\alpha}{N^{\alpha}}\frac{\mathrm{M}^{-1}}{\vartheta\left(U_c^*\right)^{-1} S\mathrm{M}^{-1}} \\ &= \frac{1-\alpha}{N^{\alpha}}\frac{1}{\vartheta\left(U_c^*\right)^{-1} S} \end{aligned}. \quad (6\text{-}18)$$

Let define $H\left(S,U_c^*\right) \equiv V_n N^{\alpha}$. Plugging this definition into Eq.(6-18) yields:

$$H\left(S,U_c^*\right) \equiv (1-\alpha)\frac{1}{S\vartheta\left(U_c^*\right)^{-1}}.$$

Notice that $H_S < 0$, $\lim_{S\to 0} H\left(S,U_c^*\right) = +\infty$ and $\lim_{S\to\infty} H\left(S,U_c^*\right) = 0$ ($g_S > 0$).

On the other hand, the market clearing Eq.(6-6) implies:

$Y = (1-\nu)S^{\nu}C + \nu\mathrm{M}^{-1}SY^*$, (6-19)

where we use $C^* = Y^*$.

Because of $C = F\left(U_c^{-1}\right)$ and Eq.(6-16), we have:

$$\begin{aligned} C &= F\left[\vartheta\left(U_c^*\right)^{-1}\omega(S)\right] \\ &= F\left[\vartheta\left(U_c^*\right)^{-1} S^{1-\nu}\mathrm{M}^{-1}\right] \end{aligned},$$

with *F* being the operator of function.

Plugging the previous expression into Eq.(6-19) yields:

$Y = (1-\nu)S^{\nu}F\left[\vartheta\left(U_c^*\right)^{-1} S^{1-\nu}\mathrm{M}^{-1}\right] + \nu SC^*$. (6-20)

Let define $J\left(S,C^*\right) \equiv (1-\nu)S^{\nu}F\left[\vartheta\left(U_c^*\right)^{-1} S^{1-\nu}\mathrm{M}^{-1}\right] + \nu SY^*$. Note that $J_S > 0$, $\lim_{S\to 0} J\left(S,C^*\right) = 0$ and $\lim_{S\to\infty} J\left(S,C^*\right) = +\infty$.

Hence, given a value for C^*, ϑ and Y^*, Eqs.(6-18) and (6-20), jointly determine the steady state value for *S* and $\omega(S)$, i.e., the steady state value of the TOT and the real exchange rate.

Dividing both sides of Eq.(6-19) by C^* yields:

$$\frac{Y}{C^*}=(1-\nu)S^{\nu}\frac{C}{C^*}+\nu\mathrm{M}^{-1}S.$$

For convenience, and without loss of generality, we can assume that initial conditions (i.e., initial distribution of wealth) are such that $\vartheta=1$ which implies that $Q=\frac{C}{C^*}$.

Plugging this condition into the previous expression yields:

$$\begin{aligned}\frac{Y}{C^*}&=(1-\nu)S^{\nu}Q+\nu S\\&=(1-\nu)S^{\nu}S^{1-\nu}\Psi+\nu S\\&=[(1-\nu)\Psi+\nu]S\end{aligned}$$

where we use a steady state condition $Q=S^{1-\nu}\Psi$ which stems from Eq.(A.1).
which can be rewritten as:

$Y=[(1-\nu)\Psi+\nu]SY^*$, (6-20)

by using $Y^*=C^*$ which is the steady state market clearing condition in the foreign country. Eq.(2-15) is no longer applicable.

Eqs.(2-17)—(2-20) is still applicable. Thus. Eq.(2-21), i.e., $S=1$ is applicable. However, even if plugging Eq.(2-21) into Eq.(6-20), we cannot obtain Eq.(2-15) because $\Psi=1$ is not applicable.

Plugging Eq.(2-21) into a steady state condition $Q=S^{1-\nu}\Psi$ yields:

$Q=\mathrm{M}^{-1}$, (6-21)

where we use Eq.(6-15). The PPP in the long run is no longer available.

Plugging Eq.(6-21) into the initial condition yields:

$C=C^*\mathrm{M}^{-1}$, (6-22)

That is, $C=C^*$ is no longer available.

Plugging Eq.(6-22) into Eq.(6-20) yields:

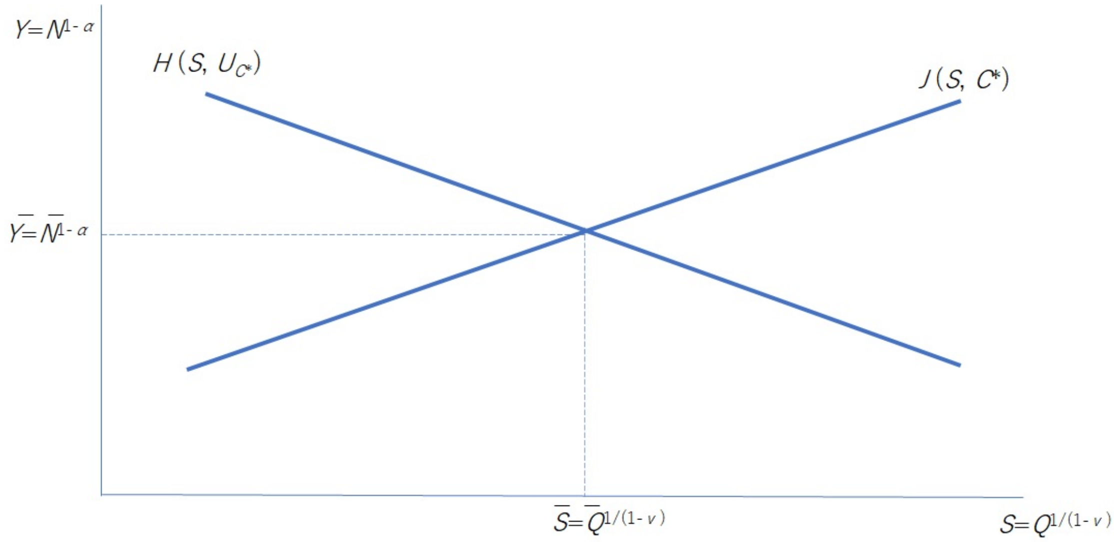
$$\begin{aligned}Y&=[(1-\nu)\mathrm{M}^{-1}+\nu]C\mathrm{M}\\&=(1-\nu)C+\nu C\mathrm{M}\\&=[(1-\nu)+\nu\mathrm{M}]C\end{aligned}.$$

Thus, $Y=C$ is no longer available.

Reference (Not shown in the text only)

Gali, Jordi (2015), ``Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Application (2nd Eds.),'' *Princeton University Press*, New York.

Figure TA--1



(Relative) PPP in a Small Open Economy in the Steady State